THE TEACHING PLAN FOR THE UNIT OF WORK IN JUNIOR HIGH SCHOOL MATHEMATICS

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J. H. Z.

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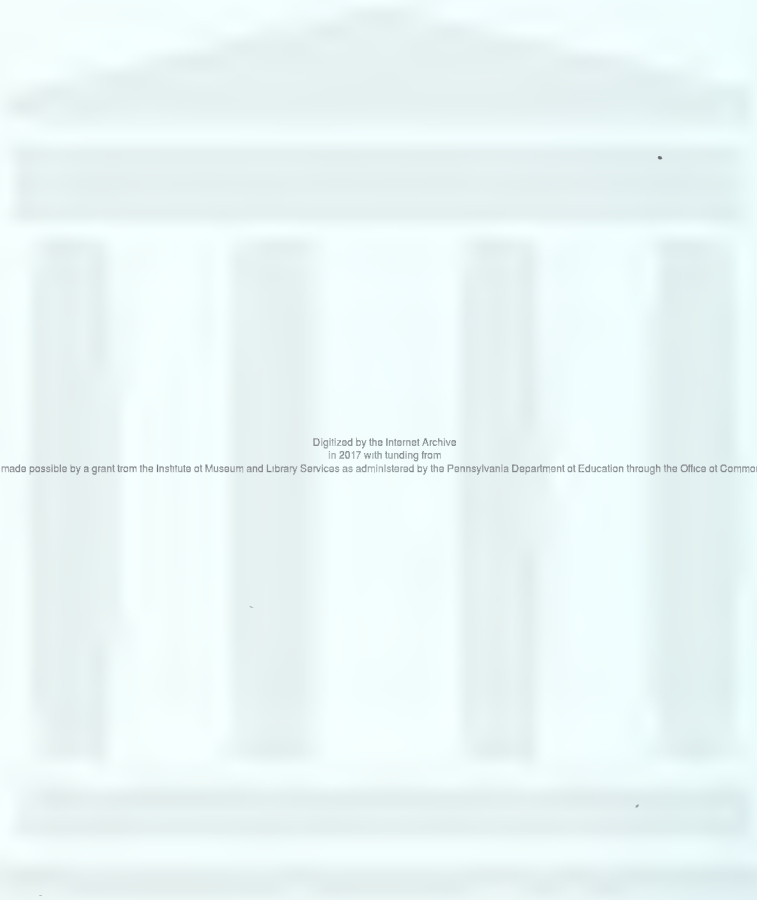

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CHAPTER I

Introduction and Definition of Terms

Purpose of Study

The purpose of this study is twofold: (1) to determine a method of improving instruction in junior high school mathematics through the use of a teaching plan of a subject matter unit; (2) to test the value of the teaching plan of a subject matter unit in actual classroom situations.

Procedure

The four steps which have been taken to accomplish the above purpose are here briefly set forth:

1. A form for the teaching plan of a subject matter unit has been built and the separate parts of it explained in detail. Any unit in mathematics may be planned according to this form. It will be necessary only to fill in each part of the plan form for the particular unit which one may wish to use.
2. The method used by experienced teachers in teaching any subject matter unit with which they are thoroughly familiar has been analyzed. The purpose of this investigation was to determine how closely this method corresponded with the proposed teaching plan of a unit. This was done by ascertaining from a large number of junior high school teachers (177) whether they had definitely written down or had in mind the separate

items of the teaching plan. These items of the teaching plan are listed and discussed in detail in Chapter II. They include among others, the aims of the unit, the method of procedure, the subject matter outline, and the tests to be given.

3. The teaching plan has been illustrated by presenting several samples from the field of junior high school mathematics. These consist of samples from each of the major divisions of the field of junior high school mathematics, informal geometry, arithmetic, algebra, indirect measurement, numerical trigonometry, and demonstrative geometry.
4. The value of the teaching plan was tested in actual classroom situations. Four units were taught, in which the teaching unit plans mentioned above were used. Approximately thirty-five teachers in eighteen public schools of Oklahoma cooperated in the experiments. The achievement of the pupils (274) of the teachers who used the plan was checked against the achievement of (668) pupils taught in the ordinary way by a like number of teachers. The same tests were used in judging the achievements of the two groups.

The Types of Units

It will be profitable to discuss at some length the unit of work with reference to its use by writers of courses of study and by educational leaders, and also the relation of the subject matter content to the units of work in both the elementary and the secondary schools. Six types of units of work have been listed by Professor Bruner. (4:1-9)

I. Units of Unplanned Experience.

"This type of unit cannot be written up in advance since it represents the expression in action of the thing in which the child is most immediately interested. There is little if any teacher guidance when this type of experience is considered in its purest form. This does not mean, however, that a clever and understanding teacher may not so manipulate environmental factors as to surround the child with a wealth of alluring situations which invite him to create and live."

II. A Unit That Starts With an Adjudged Worth-while Wholehearted Purposeful Experience and is Allowed to Eventuate into Whatever Subject Matter it will.

"The teacher's primary concern in this type of unit is in having the children involved in interesting experiences which have promise, socially and individually, experiences that will lead on to an enriched life and to a fine understanding of self and others. She is not concerned as to the field of subject matter from which the content may be drawn, but she is vitally interested in whether or not the experience engaged in will make for happy and enriched living on the part of the individual and for the improvement of society through such living. . . . . The experience may eventuate in subject matter from industrial arts, social studies, mathematics, English, or a number of other subjects. The teacher doesn't really care as long as it fully meets the criteria of social and individual promise."

III. The Theme or Generalization Unit.

"Under this head would fall the unit in which a teacher has selected a pivotal issue, theme, or generalization, the understanding of which would be most valuable for the individual and society. The first and most important task in building this type of unit consists in choosing from all the possible themes those which if understood would be most strategic in the life of the individual and would eventuate to the most good to society. Content is absolutely necessary in this type of unit, but only that content which contributes directly to the understanding of the themes and generalizations chosen . . . . .

"This is the approach which seems at the present time to offer, especially in such subjects as social studies and science, an optimum opportunity for successfully combining the two extreme notions of unbridled children's interests and adult chosen subject matter set out to be learned. The theme is chosen by adults out of the rich racial experience and after due consideration of the present and future problems with which children and adults are and may be confronted. The interests of children are elicited and held through a discriminating selection of those activities and materials which are most effective in developing an understanding of the selected theme."

IV. A Unit in Which the Teacher Determines in Advance the Informations or Skills she Thinks the Pupils Need and Consciously Plans Activities which will Eventuate in the Desired Ends.

"In units falling under this category teachers have decided in advance that pupils need certain subject matter such as the multiplication tables, spelling words, history facts, map locations, and the like, and consciously plan to lead the pupils through the kinds of activities that will eventuate in the adult desired subject matter in which the pupils are lacking. It is a type of motivation. While it is a species of sugar-coating, it is entirely legitimate, especially if resorted to after whole hearted purposeful activity has not proved successful in securing the valuable aspects of subject matter. Here the teacher's purpose is dominant, but it is the aim of a teacher who realizes that it is far more effective in the long run to capture the interest of the child than to force subject matter on him when the only reward consists of grades or teacher approval."

V. Subject Matter Units Involving Correlation.

"Under this classification will fall units devised by teachers who are more interested in subject matter than in activities or experiences but who are willing to cross subject matter boundaries in an endeavor to bring in all of the material pertinent to the topic under consideration. These units might be named units of correlated subject matter. In constructing and teaching units of this type a teacher is not limited by platoon organization or subject matter fields but feels free to draw from any field any content which will contribute to the logical development of his topic. . . . . .

"The correlation approach, however, even when it is not used to optimum effectiveness, indicates at least that the teacher has the feeling that subject matter lines are not absolutely sacred. The right type of correlation unit is one in which only that subject matter is drawn from the several fields which is approximately related to the topic being discussed."

VI. The Drill or Topic Unit.

"Under this classification will fall units developed for the sole purpose of teaching pupils certain skills and subject matter facts . . . . .

"This type of unit will have to be employed in many subjects such as geometry, and the formal portions of English, manual arts, music, and art, but should be resorted to only after every legitimate and economical attempt to start with a vital absorbing pupil interest has been exhausted."

The Use of Units by Course-of Study Writers

The older type course of study makes no mention of units.

"The old type (course of study) contains little more than bare references to existing textbooks. Its chief aim is to dictate just what pages in the text book should be covered during any period of time. The methods of teaching and learning compelled by such a course are formal and inflexible." (5:13-14)

Professor Bruner states that in 1931 "over seventy per cent of the courses of study appearing in America today are still of this type."

It can be seen, therefore, that in the majority of the elementary schools subject matter as such is still taught. This subject matter may profitably be broken into topics or units, preferably the correlated subject matter units (V above). The teaching of these subject matter units to children in such a way that they see and experience problems of modern life, and so that the creative potential within the children may be drawn out, is the problem facing the progressive teachers in these schools. It is one of the purposes of this study to suggest and to illustrate a way in which this may be done in the field of junior high school mathematics.

Professor Rugg predicts that "the next great trend in curriculum making in the elementary school is the building of an active program of meaning with experiences so rich in opportunity for reaction that literally millions of new activities get built into the life of the child." (33:27) In 1931 Professor Bruner found from an examination of outstanding courses of study in content fields, that,

1. "The almost universal tendency of the makers of the outstanding courses of study in content fields is to include a wealth of new materials and activities grouped into larger units built around central themes or big ideas . . . . . There is a distinct tendency to collect and utilize materials and suggest activities which will lead pupils, on the proper age and grade level, to a sounder realization of the pivotal issues which our country is now facing and assist them, at least in a far greater measure than in the past, in meeting the situations that arise now and probably will arise later.

2. The "tendency is toward organization of the course content on a combined topical and unit of work basis. There is undoubtedly a growing tendency to apply the philosophy of education which attempts to take account of both the child's

needs and those of adult life and requires that problems whose solutions are of permanent value be attacked." (5:14-15)

The theme or generalization unit has been used in various ways in the outstanding courses of study. Some suggest letting the unit of work make up the main activity of the pupils and bringing in only such subject matter of which the child feels the need. Other courses have definite grade objectives in the various subject matter fields, and the teacher of each grade is held responsible for having her pupils know this material. Three courses of study will be discussed.

In Allegany County, Maryland, the teachers decided that the program in the field of social studies should be undertaken first "because of the comprehensive nature" of the subject and because "of the fact that the social studies, if produced first, could then be used as the core of the curriculum. Skills in English, including reading, oral and written composition, spelling and penmanship were allocated to the various grade levels in suitable form for teachers to use in connection with content units." (1)

The following administrative principles which were listed dealt with the use of units:

"7. Production committees shall be of two kinds those concerned with the actual writing of courses of study; and those concerned with the writing of teaching units and the experimentation and appraisal of these courses in the classrooms."

"10. Before writing units or parts of any course of study, the entire program for that field of work, shall be determined so that the organization of the content may be seen in relation to themes and general objectives."

"11. It shall be the policy of the county to study the best current practices with respect to methods of procedure, content of units of work, grade placement, time allotments, etc. However in their production, the committees shall not feel limited to present practice."

The educational principles included the following:

3. The materials of instruction shall be assembled from the standpoint of the learner irrespective of the boundaries of the existing subjects."

"4. . . . . the expected outcomes of this curriculum shall be evaluated in terms of wide interests, attitudes and understandings rather than in terms of attention to isolated or unorganized parts of subject matter."

"6. No materials of instruction shall be included in the curriculum that cannot be justified as having intrinsic value for the learner." (1:10-13)

The social studies, according to the Allegany program, deal with the problems confronting people who live in social groups. The child is concerned with understanding the world about him; and as he develops, his interests spread to the more remote sections of the community, the nation, and the world at large. The child should get some ideas and understanding of how people live in other lands, the relation between the geographical environment and the lives of the people, the interdependence of people today, and the like.

The social studies in addition to interpreting relationships should develop attitudes of sympathetic understanding of other peoples, a spirit of good will and respect for other races, and the like. "The program proposes a succession of things for pupils to do - - - reading, discussing, planning, illustrating, and thinking - - - based upon the principle that a child learns by active assimilation, constant practice is given in thinking, reasoning, and drawing conclusions with the ultimate goal of an elementary understanding of life in the modern world and, in so far as time permits, a knowledge and appreciation of other civilizations." (1:14-15)

Five themes or generalizations were chosen for the program. 1. The Interdependence Theme. 2. Man's Increasing Control Over Nature Theme. 3. The Adaptation Theme. 4. Population Theme. 5. The Democracy Theme. Because these themes are abstract and difficult, they are presented through a definite body of material that is adapted to the level of interest and experience of the pupils and suitable for the development of the theme idea. The ideas or themes need to be presented many times but always through new and fresh materials. For this reason only a limited number can be developed in the elementary school. (1:18) All the different types of content

material of any unit must be focused on the theme. The understanding of the theme involves the acquiring of information, skills, attitudes, and appreciations, but these come through meaningful experiences related to the central theme rather than as isolated learnings. (1:21)

The Shorewood Course of Study in Arithmetic expresses the belief that not all of the desired ends in arithmetic will come incidentally, but the hope is that teachers will plan fruitful units of experience out of which will come most of the desired ends in every field of subject matter. However, minimum requirements for each grade are set down for which the teacher is held responsible.

"We believe the point of greatest interest for most units is in the social sciences, hence we have suggested only a few strictly arithmetical units here, believing that in most correlations the arithmetic values will have to be 'pulled out' of other units or deliberately taught as drill matter." (40:12)

"Units chiefly valuable from the arithmetical viewpoint are very hard to find . . . . . Most of them are really based on social science and should be. However, even here there are many arithmetical values which can be pulled out if the teacher is watchful of her opportunity." (40:82)

The unit in this course of study is ideally one that permeates the whole school activity. It is not necessarily, however, all that the child does during any one week or month. In case some desirable skill is being neglected the teacher is supposed so to arrange his work, either within the unit or out, that this desirable skill may be properly taught. Since the course of study lists the required skills in the various subjects, the teacher can easily check at any time.

The Denver Colorado Course of Study states:

"Many authorities on curriculum making agree that all subject matter of measurable value will be included in activities that are vital, and that any subject matter that serves no purpose whatever in a desirable activity has little value since its inclusion would make no difference in the direction or control of behavior. For this reason the attainments or growths in this course have been set up as actual needs rather than as subject matter requirements. The thoughtful teacher will prepare

the situation so that as many subject matter needs and experiences as possible will grow out of it." (7:2)

The course of study lists a large number of minor activities with possible child growths. Later major activities or units of work are illustrated. No definite subject matter aims as such are set up, and no work other than the large units is recommended. This bulletin deals only with the kindergarten and primary grades.

The Lincoln School of Teachers College is a twelve-grade-laboratory school, established to serve as "a laboratory for curriculum experimentation", and as such has its regular research department. The teachers have done a large amount of experimentation with units of work. Many of these have appeared in book form as accounts of total classroom experiences written by the teachers of the units. These experiments cover work from the kindergarten through the tenth grade.

The main characteristics of curriculum making in the Lincoln School are:

"First, the curriculum is organized around units of work based on genuine interests of the children; second, the major responsibility for the selection, organization, development, and teaching of the unit rests on the classroom teacher; third, the unit is planned in broad general outlines previous to teaching; fourth, the process of planning is continuous until the unit is complete; fifth, a record of development of the unit is made; sixth, the teachers of special subjects act as councilors and advisors to the classroom teachers; seventh, the curriculum research staff offers aid throughout the entire process, and, in addition, locates and studies special curriculum problems." (14:414)

The objectives of the teachers and pupils may be grouped into four broad divisions; first, skills, habits, or specific learnings, which may range from learning a specific fact like an addition combination to the development of the power to control emotions; second, concepts, ideas, or understandings; third, generalizations, principles, or rules; fourth, attitudes, appreciations, or dispositions. "While the teacher is generally concerned with the growth of the children in specific learnings, she is more anxious about their general habits of work, their

growing power to attaek problems.'' She does not neglect facts but is more interested in meanings associated with these faets. Her main interest is in the fourth division above, sinee attitudes and appreciations will lead to skills, coneepts, and generalizations. (14:412)

This larger unit is often spoken of as the integrated unit. Integration may be thought of as ''the proeess of organizing experiences into a unified whole or generalization in order to enable the individual to make appropriate responses.''* In the integrated unit it is one of the aims of the teaeher so to organize and teaeh the unit that the pupils may experience the process of integration.

This idea is not new in the educational field. The proponents of the integrated unit and the integrated eurrieulum hope to attain this end by their particular organization and use of child activities. Many others have sought the same ends by other forms of organization. One of these is known as the correlation of subjeet matter. The earliest experimentation with correlation was in the field of junior high sehool mathematics, but the first suecessful reorganization eventuated in the general seienee eourse for junior high sehool grades. This topic will be diseussed more fully later in this chapter.

In the lower grades of the Lincoln School the units of work compose the main aetivity of the elass, and the materials are so integrated that the children get the desirable skills and learnings. (15) (8) This usually eontinues through the third or fourth grades. As the children get older, however, periods are set aside in whieh to learn skills and informations. The unit of work still oecupies most of the day, but there is teaching and drill on other items; some are motivated directly by the main activity, and some are assigned by the teaeher. Usually, however, this is not neeessary. One teacher says, ''At no time did the teaeher attempt to draw out of the unit of work artificial situations in order to stimulate drill. The faet that children need and feel the need of some degree of facility in these skills was suffieient reason for devoting time and effort to their mastery.'' (17:56) In the junior high sehool the units

* Virginia State Board of Education Bulletin. XV:49.

of work have been limited to certain subjects, while other subjects were taught in the usual way. (49) The units of work involving subject matter from a limited number of fields have also been tried in the high school division, but reports on it are not yet available.

The Horace Mann School of Teachers College "attempts to develop in each of its children four fundamental powers," *the power to know, do, think, and feel.* Four areas of education are used in developing these powers in the children, the area of speech, the area of mathematics and science, the area of tools, and the area of human relationships. (31:3-5)

The second part of the plan lists the educational outcomes and core of content in the belief that "a rich and many sided personality possesses information." The statement of the core of content has several purposes: 1. In justice to the child the work of any year must be based on previous training and on what will be expected of him in the future; 2. In justice to the teacher there should be a definite statement of the content for which she will be held responsible; 3. By means of the core of content the concept of relationship can be brought to the teacher and to the child. "In its total form it presents not a complete educational experience for children but rather a statement *for today* of certain educational outcomes which the Horace Mann Staff thinks all of its children should possess." (31:17)

"Teaching units have been defined as follows: materials, experiences, situations, and methods of presentation arranged for teachers around centers which are of interest to the children." These centers of interest and teaching units are merely methods of teaching. The child satisfies a natural curiosity but at the same time comes into possession of desirable facts and skills. In developing these units the child usually goes far beyond the requirements of the core of content, and "his individual needs and possibilities can be cared for and developed." (31:18)

The teaching units in the Horace Mann School often make up the main school activity of the class but probably do not as a rule continue over as long a period of time as in the Lin-

coln School, where a class studies a single unit for an entire year. Core content is more completely set down and allocated as to grades and teachers, and for this reason probably more of the formal skills and informations are taught during any period of time.

The Place of Subject Matter Content in the Integrated Unit.

The subject matter content is taken to mean the skills and informations of such subjects as arithmetic, science, English, and the like. Those who are advocating the integrated unit or the integrated curriculum by no means ignore these subjects. Much has been said above about the subject matter skills and informations. However, it is useful to examine more closely and see just what happens to them.

In setting the child as the center of the curriculum and using his interests as the basis of the things to be studied, the subject matter becomes purely a means to an end; and if the philosophy is carried to its logical conclusion, only the subject matter in which the child is interested will be used. This subject matter may not even bear on the unit or theme under discussion but may be merely sporadic interests entirely apart from the unit. Nevertheless, they are child interests and deserve a place in the curriculum.

In the published reports of the units of work, however, few of these sporadic interests of children are alluded to. A skillful teacher can keep the children working toward desired ends. This may be done by timely suggestions to the pupils or by appealing to the group as a whole as to the relevancy of a proposed activity. It means teacher guidance to a certain extent.

An examination of a large number of courses of study, and also integrated units prepared independently, reveal that, while the authors may subscribe theoretically to the idea of using only the subject matter that interests children, a feeling of compulsion exists which forces them to make provisions for the attainment of certain skills and informations by the pupils. This idea may be due to their own experience in teaching the older type curriculum, or it may be due to certain outside pres-

sures. Certain people, sometimes the children themselves, still expect the teacher to teach certain types of subject matter in the schools.

Many individual teachers have published accounts of their use of certain integrated units. The Lincoln School Series, Units of Work, is an example. A number of these teachers frankly admit that they guide the choice of activities of their pupils to fit the desires of the teachers, and they also admit that there are certain desirable skills and informations which the pupil should have. These skills and informations will not necessarily come from the activities of the particular class in pursuing the unit. In the Horace Mann Elementary School the things which the pupils are expected to know at each grade level are definitely set down in the Core of Content discussed above. (31:15-17)

The following quotations are from the published reports of the units of work.

"The teacher should not hesitate to initiate some of the work herself and must not follow unimportant leads of the children." (17:12)

"However, this incidental arithmetic does not provide sufficient mastery and the teacher is continually conscious of the fact that she must set aside time for fixation of many of these number facts and skills." (17:249)

"On the other hand the indiscriminate following of the children's leads and interests tend to make for chaos." (49:18)

"Certainly there are materials and activities worth pursuing in and of themselves and little or no advantage could be gained through integration." (49:243)

The teacher "sees that there is a related and integrated development of subject matter and growth in the mastery of skills." (15:7)

"The children were aware that drill in and knowledge of certain new operations in mathematics were included in the school curriculum for the fourth grade." (8:13)

1. "If we are going to organize activities around children's spontaneous interests, if we are really going to let the children feel the need of each thing they learn we are bound to leave unfelt the needs for a good many things the children will need

later in life. Therefore, our curriculum is not going to be complete.''

2. The project method does ''not make sufficient adaptation to individual differences.'' A condition arises in the activity where short division is needed, but not every child is equally ready to study it, and each child would require a different amount of drill.

3. Because ''the teacher realizes that the child is eventually going to need some of these things for which he does not spontaneously feel a present need, she . . . . . begins to cook up the project so that it will make the child see the need for something he is going to need some day, and the project begins to lose its spontaneity, and its naturalness. It gets warped and twisted through the attempt to drag in some foreign subject matter.

''On the other hand the teacher does not want to go too far in this direction and therefore also fails to include some necessary subject matter.'' (58:31-32)

An examination of the results of standardized tests given in connection with the Lincoln School Units of Work shows that in all cases the grade gains for the year were higher in reading than in arithmetic and in all cases except one higher in reading than in spelling. It should be said, however, that the classes were well above the normal score in each of these subjects. They were also well above the average in intelligence. The grade gain in reading ranged from .9 to 2.2, in spelling from .4 to 1.6, and in arithmetic from .4 to 1.3. This result is to be expected, however, since nearly all activities in the units used involve reading to a much larger extent than the other subjects. The teachers were not concerned with teaching any large amount of the skills and informations but spent most of their time trying to attain other objectives, such as good habits of work, ability to think and generalize, and the like.

The use of the integrated unit has certain dangers which should be pointed out. The results obtained by the use of this method depend, as in other methods of teaching, on the type of teacher using the method. First, an inexperienced teacher has no rich background in teaching and in the subject matter to be taught, and probably he will not consider the subject matter valuable and neglect it more than the older and better

trained teacher. This is a problem of teacher training; the young prospective teacher must know the content well because even though he is not to attempt to include it in the curriculum, he must be able to select. In the other type of teaching it is selected and organized by the textbook writer. Second, the teacher who has little preparation, whether he is experienced or inexperienced, will have neither the pedagogical basis for the selection of the subject matter, skills, and the like nor a wide knowledge of the materials of the subject from which the selection is to be made. Third, certain experienced teachers will be without any adequate preparation in some of the subjects. This is more especially true in the junior high school and high school grades. In these grades the teachers have taught or prepared to teach certain subjects like English or history. If they are required to develop and teach an integrated unit, some subjects will be neglected while others will be over stressed. Centering the entire curriculum around a social science theme is likely to lead to the neglect of some necessary skills, informations, and appreciations in mathematics or some other field, merely because the teacher does not know enough about these subjects to see all the significant relationships. This is probably the most serious fault of this method of teaching. Fourth, the advisors and directors of curriculum revision programs may have little knowledge of or sympathy with the aims of the separate subjects.

The fact that teachers are poorly prepared to use an integrated unit, however, is no reason for refusing to advocate the method. The point is that teachers must be properly prepared to do such work, if it is to be successful.

There is some disagreement as to the value of the integrated curriculum as compared to separate subjects as modern teachers would teach them. Reeder, (28) who proposes the title "subject matter modernist", finds "a very large body of theory which is common to both points of view." Both concepts of teaching have as their central idea "the respect for children, which demands their happiness, childlikeness in activity, and reasonable freedom." Both groups agree on the concept of learning as an active rather than a passive process." Both groups accept the importance of child purposing and planning."

"The core of the whole argument is the question of organization of subject matter." The "separate subject modernist" offers a group of subjects that have come into existence, "not by some whim or fancy of mankind, but because man found that if he organized his knowledge of individual facts into a functional whole, which possessed a usable point of view", he could remember better and could have a workable equipment of facts and skills to bring to bear on a particular situation in his experience. This is in line with the viewpoint that the teacher should definitely aim to teach the subject matter that is valuable to the child and to teach it as a definite body of connected material. Otherwise, the pupil may not get it at all, for as Professor Breslich has recently said, "*Incidental* teaching is pretty sure to be *accidental* teaching." There is a unity of thought in each of these subjects of study, which is expressed in the aims of each of them. For example, the purpose of arithmetic is to help the child deal efficiently with the quantitative relationships of life . . . . . .

"Each of the . . . . . statements of aims of the elementary school subjects constitutes a real guiding principle both in the selection of material and in the direction of teaching and study."

The integrationist presents units on "such topics as food, water, transportation, architecture," and the like. "The great difficulty about such units is that they really belie the name and usually possess no real unity of point of view, ... no essential coherence of thought." The facts presented or studied in these units have a thread of connection through the general theme, but it is far from a real unity of the facts as is found in a subject matter unit.

Both groups admit that the school room situation under the separate subject plan does not correspond to real life, but the "subject matter modernist" contends that one can meet a real situation better if he has a well organized body of functional knowledge in the subject matter fields. The "subject matter modernist" also believes in using as much real correlation between subjects as possible.

The Integrated Unit of Work in the High School.

Dr. Will French (11:134-135) finds from a study of twenty-two high schools, chosen by "fourteen experienced men of high professional standing in education" as "representative of advanced curriculum practice," that Morrison's *Practice of Teaching in the Secondary School* has had a wide influence in all parts of the country. This has led to "an enlarged and more comprehensive idea of the term unit." The emphasis on the assignment has shifted the center of work from the teacher to the pupil, and schools coming under this influence have had larger reading lists for the pupil.

The "overview" has had a good influence in general, for it gives the pupil a better chance to integrate the whole unit. However, in general there is a greater change in methods of teaching than in content. The rapid multiplication of courses in high schools shows little change in content; it is merely old subject matter taught under a new label.

Dr. French suggests that "much of our present advanced practice is but a superficial attack on the whole problem of revision of the curriculum of secondary schools for

1. it has assumed the value of traditional content
2. it has clung to a logical organization of subject matter 'set-out-to-be-learned'
3. it has accepted departmentalization of learning as necessary
4. there has been no critical evaluation of practice in terms of worthy purposes of education
5. the interests and immediate needs of the pupils have been given too little consideration."

He suggests a fundamental reorganization of the secondary school curriculum which will lead to larger units than we now have, each focused on one of the objectives of secondary education. Each of these objectives implies attitudes and skills of importance in the best social living, and each of these attitudes and skills in turn is "underlaid by a whole hierarchy of smaller but similar and supporting understandings and skills." Each unit would be focused on an objective of education, and only the subject matter needed would be used. Both pupils and teachers would understand that the subject matter is a means,

and that it would be worthless to gain the subject matter without achieving the objective. This may be brought about by making sure that the unit test is in terms of "understanding of, attitudes toward, or skills in the objectives of the unit, not in the subject matter of the unit."

"These units when built should be assigned to the department and teacher who, by reason of training or experience, has a good command of, respect for, or interest in the objective being stressed by the unit. Usually this will mean one who is in command of most of the subject matter involved. There should be no omissions of subject matter, however, to fit into departments or teacher knowledge . . . . . To make up for any deficiencies in teachers' training, a teachers' handbook to accompany the unit should be prepared."

"Before these units can be well built, educators—principals, superintendents, college of education professors—should formulate the broadly conceived understandings, attitudes, or skills which underlie the stated objectives of secondary educations and also those for the smaller, supporting unit objectives. These committees of teachers drawn from several present departments should take up the task of writing the unit."

Many people, including those interested in general education as well as those interested in some particular subject, would not agree with all of the ideas expressed by Dr. French. For example, advanced practice in the teaching of mathematics does not assume "the value of traditional subject matter." Nor will specialists in this field agree that "there has been no critical evaluation of practice in terms of worthy purposes of education." Like statements could undoubtedly be made regarding other subjects. Dr. French makes the common mistake of choosing the worst practice in the older type of curriculum and comparing it with the best in the new.

His method of dealing with the subject matter needed in the development of the units is weak. To take the ordinary teacher of social science or any other subject and hope to make up his deficiency in an unfamiliar field, like mathematics, by the use of a teachers' handbook is to overlook the importance

of scholarship altogether. Teachers, as a whole, know too little about the subjects in which they are certificated to teach.

Dean Stout proposes a method which is more likely to obtain desirable results than the one discussed above.

"A movement is now in prospect in the secondary school to attempt the organization of at least a few *functional units* which will ignore subject and even field limitations . . . . . The teaching and learning of subjects or fields as such, still dominates educational practice in spite of all that has been attempted to secure revision and reorganization of curricula.

"Organization of one such unit or even two corresponding both in name and purpose to each of the four objectives of secondary education (health, leisure, vocation, and social relationships) will permit continuance of one or more subject units in each of the traditional fields of instruction. The former will provide the common elements in curricula and furnish the means for a liberal or general education while the latter may serve in supplying, in part at least, the differentiated ones required for college entrance and later specialization in the traditional subjects and fields." (47:114-115)

Methods similar to this have been adopted in some schools, but few reports as to the actual results have been made available.

Dr. Newlon, after briefly reviewing the progress of the idea of integration in the elementary school and pointing out instances in which it had been used in college, says

"The senior high school has been the last unit in the educational system to be touched by this movement, and is as yet little affected. But a new interest in this approach to curriculum construction is clearly discernible in the high school."

Dr. Newlon also points out the fact that there are many values of the curriculum that should be made available to boys and girls but which are impossible so long as the present type of curriculum is used. For example, "home economics courses with their attention to health problems, to the home, and to the economics of sociology of the family involve many values to the education of either boys or girls in the modern world." These values may be made available to all only by the introduction of courses which will cut across subject-matter fields. Undoubtedly there will also be available in the secondary school

subject electives, "but these subjects while of indispensable social and cultural value *will be* electives. [as they are now in mathematics] . . . . . In brief the integrated courses should provide more effectively many of the values of general education, while the subject electives, especially in the upper years of the secondary school, should provide for the beginnings of specialization."

The outline of the proposed bulletin, "Scope and Sequence of the Curriculum for Virginia Secondary Schools," (to be published late in 1933) recommends for each year of the four-year high school eight or more *Functional Phases of Social Life* with the corresponding centers of interest. The suggested units of work for each big topic may lead to the use of the subject matter from various fields, social studies, language arts, natural science, home economics, physical education and health, agriculture, mathematics, and guidance. During the first year it is suggested that all of these fields be used at various times, though it would be possible for the pupils and teacher to select units that would exclude some particular subject for the entire year.

For the other three years of high school the suggested subject matter for the units does not include any mathematics and very little from the fields other than from the social studies and language arts.

The teachers in each subject matter field were asked to cooperate, and they made every effort to do so, but the teachers of mathematics felt that their subject was of such a nature that it was impossible to attain the subject matter aims through the use of material of this sort. It may gradually be worked out, however.

If the integrated unit is to be acceptable to secondary teachers and administrators, it will have to be worked out by people who are interested in the different subjects, and who are familiar enough with the subject matter in order to be able to see the relations of the various parts. Ideally a person developing or teaching such a unit should be familiar with much of all the subjects involved. To be able to see relationships properly, he should know much more about the subjects than any-

one would expect to teach. In high school subjects it would mean that one would almost have to specialize in many different subjects, such as English, mathematics, social science, art, and the like. According to one principal "it is hard to get teachers who themselves possess the knowledge which underlies the skills they propose to teach." (11:131) Those who are directing such a program should also have a wide knowledge and broad sympathies.

The type of integration referred to above is ordinarily taken to mean integration between two or more different subjects or fields. It might be called inter-subject integration. The integrating factor, or the thing which controls the selection of class activities, is usually some theme or big idea. This study, however, deals with units of work which illustrate another type of integration, that is, integration within the great field of mathematics. Here the aims of the unit and of mathematics in general are the integrating factors. Both types of units have the same end product in view, a well-educated individual, capable of living usefully and happily in society. The units used in this study are built on the present interests and abilities of the children, and any sort of subject matter which contributes to the central interests or aims of the units may be freely used. They may be called coordinated subject matter units. (No. V above).

This intra-subject integration has been used for a number of years, especially in the junior high school field.

"The earliest experimentation was with mathematics, but the first successful attempt at unification or integration was in general science. It was felt that youth of the junior-high-school age should have a practical and interesting introduction to science. The most successful general science courses were built around scientific problems closely connected with every day experience of children that involved not one science but several. Many conventional-minded science teachers considered such a program utterly unthinkable, but the new idea had a strong popular appeal and in twenty years has certainly demonstrated its validity. General science courses are common now in secondary schools throughout the entire country. About a decade ago this principle was introduced in the social sciences, particularly at the junior-high-school level. So called "unified"

social science courses, organized around broad themes or problems of contemporary life according to various plans, gained widespread acceptance, and it would look as if history were about to repeat itself and the unified social science programs become as popular as the general science offerings. In recent years a more satisfactory integration of mathematics subjects has been effected at the junior-high-school level, and the general mathematics course is now the accepted practice in all schools that make any pretense to being progressive. This tendency is now discernible in the offerings of a number of senior high schools.'' (24:397-398)

These subject matter units also involve correlation with other subjects and with life outside. One of the aims of such units or courses is to give the pupil a general but definite view of the field organized as ''a functional whole'' and possessing a usable point of view. The pupil would then have a workable equipment of facts and skills to bring to bear on a particular situation in his experience.

However, the primary purpose of this study is not to propose a definite type of unit in mathematics, but to propose and test a method of teaching to children units which are already in existence in courses of study. This study holds no brief for a particular way of organizing subject matter, but is primarily interested in constructing a teaching plan of a subject matter unit already organized. The teaching plans which are included later in the study are given for the purpose of illustrating how the plan may be used with different types of units in junior high school mathematics. The particular teaching unit or its organization is secondary, though it is desirable that the unit be well organized and teachable.

Certain characteristics of the ideal subject matter unit, as the term is used in this study, are listed for purposes of comparison and completeness. These characteristics are useful in constructing a plan form (discussed in Chapter II) for any unit and also in constructing a subject matter unit for a course of study. These characteristics of a ''correlated unit of subject matter'' are not based on objective evidence, but were listed after reference was made to a large number of books and articles on curriculum making. They should therefore, be looked upon as the definition of the subject matter unit.

Characteristics of a Good Teaching Unit

1. There should be a definite center of interest, or a centralizing idea, about which the work of the unit may be organized. In the selection of this center of interest, careful consideration should be given to the probable attitude of the child toward the unit, as well as to the practical value of the material to be taught.
2. The unit should "involve a variety of activity, or behavior, including answering questions, observing demonstrations, discussing, preparing materials, comparing processes and results, and the like."* (12:181)
3. All the activities of the class period should be made to contribute to the center of interest.
4. Any type of subject matter or material may be used which will help to develop this central idea.
5. The unit should state clearly where this subject matter or material may be obtained.
6. Each teaching unit should be a part of a connected whole, which should represent the work in the subject for some given period.
7. Each unit must be broad and deep enough to furnish interest and work for the best members of the class and yet simple enough that the majority can get the essential points of it.
8. Each teaching unit should help to meet the social needs of the pupils and to clarify social meanings.
9. Each unit must be capable of being so taught that the pupil will be aware of and appreciate each of the above items.

Teaching Units in Junior High School Mathematics

Until the beginning of the junior high school movement the mathematics included in the junior high school was in nearly all cases arithmetic in the seventh and eighth grades and pure algebra in the ninth. The organization in practically all cases was strictly logical, and thus the teaching units, if they could be called such, were logical units. It seems probable that in the majority of cases the pupils studied the material from day to day, recited it, and passed on, and were only rarely aware of the connections which the lessons they learned bore to the

* From Henry Harup, The Technique of Curriculum Making. By permission of The Macmillan Company, publishers.

whole topic, to each other, or to life and living outside the school room.

With the necessity for reorganization, emphasized by the junior high school movement, teachers of mathematics for these grades began to look at the subject as a whole, and as a result began to think of relating all parts of the subject matter to the central objectives of the course. Consideration of units which would contribute to these ends was the natural result. Such parts of the subject matter may not be called teaching units. They may be poorly organized and include many non-essentials, but they contain, however, the possible characteristics of a teaching unit.

It might be added that some of the experimental schools began this reorganization of mathematics some time before the first mention of the junior high school. Thus, the teachers of mathematics of the University High School at the University of Chicago were teaching reorganized mathematics as early as 1903; they were probably among the first to designate the parts of the subject matter taught as units of work or teaching units. (22:57-64)

The Teaching Plan of a Subject Matter Unit

In order to define the teaching plan of a unit we shall look a little more carefully at the unit of work. In one sense the unit is a unified way of organizing subject matter and, of course, presupposes that the outcomes with the children will be affected, perhaps dominated, by the organization. It can be called a teaching plan of the unit, for the writer of any text book or unit certainly has a definite teaching plan in mind. The teacher of the unit seldom has the same definite plan in mind, however, especially if he is inexperienced. The organization of the unit has little effect on the attitudes and appreciations developed in the children. These must be definitely worked for by the teacher if they are to be attained.

Many of the defects incident to poor teaching as those mentioned above, are remedied with experience. If the teacher is always on the lookout for something new and better, as he goes over the topic again and again, its meaning becomes clearer to him, ways and means and devices of teaching particular parts

of the unit so that the children will acquire the aimed for skills, attitudes, and appreciations will be gradually accumulated, things that will be more helpful in gaining desired outcomes with the pupils will become more apparent, and the like. But can such a thing come only with experience? It is the purpose of this study to show that a more careful plan made out by the teacher or by someone else will materially aid him in his work.

The teaching plan of a unit means something more than the textbook writer's outline. Many things are included in the teaching scheme of the artist teacher that will not appear in the textbook or in the teaching unit. Three of these will be discussed briefly as illustrations. The first is the unity and extent of the topic being taught. The teacher knows about it as a whole; he also knows much more than he intends to present to the class. The first part can be obtained from the textbook, but the other cannot; it must be secured elsewhere. The second is a knowledge of his pupils, their social environment, their mental ability, and their aptitudes in this particular topic, and the like. If this knowledge of the pupil is necessary as a part of the general plan, all the teachers will be more likely to obtain this information than they would otherwise. The third is a definite testing program for the unit. Few teachers have a definite testing program, and the failure to test on teaching is probably one of the weakest points in their teaching. These tests should be used both as teaching devices and as methods of checking the knowledge gained by the pupils.

CHAPTER II

The Form of the Teaching Plan of a Subject Matter Unit for Junior High School Mathematics

It is the purpose of this chapter to present the form of the teaching plan for a unit in junior high school mathematics, to state how it was derived, and to give a detailed explanation of its separate parts. The chapter will also present the results of a study carried on to show to what extent experienced teachers have made use of the separate items which make up the teaching plan.

The form of the plan presented here has been adapted from a form which was used in a curriculum study begun in Southeastern Oklahoma, sponsored by the Southeastern State Teachers College, with Professor H. B. Bruner of Teachers College, Columbia University as advisor. The plan has been changed in many points, but the main outline is much the same. Special features have also been taken from one of the plans in use at the Curriculum Construction Laboratory of Teachers College. The Teaching Plan Form is reproduced below.

The Teaching Plan Form for a Subject Matter Unit.

Section I. Identification

Teacher:_____ Grade:_____ Date:_____

School:_____ Subject:_____

Title:_____ Duration:_____

Section II. Subject Matter Outline

Section III. Pupil Inventory

A. Inventory items	No. of pupils E : VG : G : F : P
1. General intelligence	: : : : :
2. Previous knowledge of the unit	: : : : :
3. General social background	: : : : :

Note: E=excellent, VG=very good, G=good, F=fair, and P=poor.

B. Occupation of parents — No. of pupils
   1. Day laborers, domestic help, etc. _____
   2. Artisans, mechanics, carpenters, etc. _____
   3. Lawyers, doctors, farm owners, etc. _____

Section IV. Aims or Objectives
- A. General aims
- B. Specific aims
  1. Knowledge
  2. Skills
  3. Attitudes or appreciations

Section V. Suggested Activities and Exercises
- A. Activities for the class or parts of the class
- B. Exercises
  1. For use in regular class work
  2. For use as supplementary exercises to care for individual differences
     a. For remedial purposes
     b. For better group of pupils

Section VI. Teacher Procedure and Means
- A. The use of the teaching unit plan during teaching
- B. The detailed procedure in teaching the unit
- C. The use of the supplementary material

Section VII. Correlations with Other Subjects
- A. With other subjects
- B. Within the subject
- C. Within the unit

Section VIII. Bibliography
- A. For teachers
- B. For pupils

Section IX. Testing Program
- A. Tests to determine the pupils' previous knowledge of the unit
- B. Tests to be used as teaching devices
- C. Tests to determine the pupils' knowledge of the unit.

Section I. Identification. This section merely calls for certain items of information, as the name of the teacher and school, the title of the unit, and the like. Hence it will not be necessary to discuss it here.

Section II. The Subject Matter Outline should usually follow in general form the material in the textbook, since the pupils are using this book continually. This does not mean that a plan must be made to follow a unit in a poorly written text-

book exeept for specifie reasons, nor does it mean that the teacher should hesitate to add to the material in the book if it improves the unit from the standpoint of pupil learning, or if the material is made to fit local eonditions better. These additions and omissions will usually serve the pupil better if they include only the smaller parts of the unit. It would probably be wise to inelude in this seetion an outline of the material mentioned in Section V, B, 2 as supplementary problems; mueh of this, of eourse would be taken from other books.

A pedagogically organized unit in junior high school mathematies is not usually presented as a strietly logical discourse. It is quite possible that the teaeher in considering the group of pupils which he has to teach will decide that some other method of arrangement will fit the children's purposes better. Teachers should remember that they are not required to follow the outline given. It is presented to give the scope of the subject matter available. This subjeet matter may be used if the needs of the problem, the situation or the pupils demand.

The inclusion of the subject matter outline in the teaching plan is justified from at least two standpoints. First, it should aid greatly in giving the teaeher the unitary idea of the topic. Making such an outline would foree the teaeher to look at the unit as a whole. Having an outline available in a teaching unit plan would make it much easier for him to present the unitary idea to the pupils. Seeond, the subject matter outline of the material to be taught should help the teacher to eliminate obsolete material.

Section III. Pupil Inventory. It is hardly possible that a complete inventory will be seeured for every pupil. The teacher should, however, know something fairly definite about the social and eeonomie status of each pupil, his mental level, his opportunities for home study, the available reading material in the home or otherwise aceessible, and the like. It is probable that reading material in the homc would have little value for certain of the so ealled formal units in junior high school mathematies. However, if a teaeher is eolleeting information about pupils, it would be wise to make one visit or interview do. If the information is not needed for one particular unit, it will

probably be needed soon for another one. Reading material in the home also gives some indication of the attitude of the parents toward the child. In the larger school systems or others where fairly complete records are assembled for each pupil, much of this information can be secured with little effort, but where the records do not exist the teacher will have to get them by visiting the homes of the pupils, by giving tests, and the like. A personal informal private talk with each pupil during the first month of school would help to develop better personal relationships between teacher and pupil.

Another important thing that the teacher should know about the pupils is their previous knowledge of the unit to be taught. This is probably more valuable to the teacher of mathematics than to any other. For this purpose a preliminary test on the unit should be given at the beginning. This will be called the pre-test, and should be included as part of the Testing Program in Section IX.

The remaining items in the inventory deal with the general social background of the pupils. The teacher will have to get them by means of contacts with the children and with their parents or with other teachers or people who know well the condition of the community. Some of the specific things about which the teacher must know are: the general home conditions; the occupation and culture of the parents; the general community conditions; and any special conditions which will affect the pupils' actions, as, for example, non-English speaking parents, foreign birth, attendance at special types of schools, such as government or private Indian schools, and the like. Of course if these things are once collected and organized for a group of pupils, it is not necessary to make many changes from one unit to another. We must recognize also that not all teachers will be qualified to make desirable decisions on all the items listed in the pupil inventory. This will be true especially in regard to the culture and home conditions of the parents. The knowledge revealed by the pretest will of course vary from one subject or unit to another.

Methods of teaching, materials to be used, standards at which to aim, and many other things depend upon the results

revealed by such a study of the members of a class as outlined above. For this reason the pupil inventory is placed near the first of the plan, and many of the special aims, methods, activities, problems, and the like, are discussed in reference to groups of pupils, who have been isolated by this procedure.

Section IV. Aims or Objectives. In this teaching plan the aims are divided into two classes, general and specific. Certain objectives are of a general nature and may pertain to other subjects. These objectives may even be attained without teaching any of the subjects in the ordinary school curriculum.

It seems reasonable to assume that these general aims may be attained, in part at least, by the study of mathematics. The question of the transfer of the powers, acquired when these aims are attained, "to life situations depends upon the teacher; if such transfer is sought it is easily secured." (44:37) A single teaching unit may contribute only a small part to one of these general aims, but the cumulative effect will be sufficient if the teacher definitely seeks to attain the aims in each unit taught. These general aims may be (1) exercise in certain habits, such as neatness and method, checking results, power to think or concentrate, and the like; (2) exercise in fundamental modes of thought, such as simplicity of language, accuracy in reasoning, originality in thought, and the like; (3) to assist pupils in setting up interesting and attainable aims for themselves.

From the standpoint of a unit the general aim also includes those items which the unit will contribute to the mathematical objectives of the year or course. Thus in teaching a unit on insurance in the junior high school one of the general aims may be to interest the pupils in the social values of arithmetic, or to develop the habit and power of quantitative thinking. This is fully illustrated in the teaching unit plans included in Chapter V.

Specific aims may be thought of as the direct means used in attaining the general aims. Thus certain skills and certain items of knowledge are taught the pupils in such a way that the pupils acquire habits of neatness and checking, of learning

to think with simplicity and precision, and of seeing the social values of arithmetic as a whole. The specific aims are divided into three parts: items of knowledge, skills, and attitudes and appreciations. These are the outcomes that the teacher will actually be seeking to achieve from day to day in the classroom. Whether the transfer is made to the more general things mentioned above or not will depend on the skill of the teacher in teaching the specific items required in the course of study and upon his ability to coordinate them in such a way that the elements will transfer to another situation. Definite suggestions as to how this may be done are included in Section VI, Teacher Procedure and Means.

It is probably not necessary to make any extended effort to justify the necessity for the selection of the aims for any teaching we are to do, but only to discuss some of the advantages of selecting and classifying them as called for in this plan.

1. The aim sets the goal for the teaching efforts. The fact that the teacher is required to write out the aims tends to fix them in mind and make them function in the maximum way. If the aims are divided into two classes, general and specific, the teacher will probably have a clearer idea of their meaning and will be able to use them more understandingly. In mathematics, in particular, the specific aims will be more valuable if divided into items of knowledge, skills, and attitudes or appreciations. This will give the teacher definite ideas about teaching any unit. It will be necessary, for example, to teach each one of the desirable skills separately and carefully. Having these definitely selected beforehand will probably do much more to insure the careful teaching of all of these things than anything else.

2. Another very important use of the aims of teaching a unit is in making and selecting the tests. To be valid a test should not include any items that have not been taught to the class. This can hardly be done unless the aims are thought out carefully beforehand. The subject of tests will be more fully discussed below.

3. A preliminary selection of the aims will be a great help both to the teacher and the pupil, in coordinating and putting proper emphasis on the parts of the unit.

4. In a like manner the non-essentials are much more easily recognized. If the teacher knows just what he expects of the children when the unit is finished, he is much more likely to know what things will fail to contribute to these ends. To be able to recognize non-essentials is of particular importance in the field of mathematics. There are so many non-essentials in the average course of study that it is very easy to introduce isolated topics which contribute little to the aims of the teaching unit.

5. The mere thinking of these aims is not sufficient, however, especially for the young and inexperienced teacher. The four points mentioned above would probably be considered sound by the majority of teachers. The writing down of these aims is not so universally accepted, however. It is the justification of this procedure that we are most interested in in the discussion of Section IV. In the rush of teaching it is easy to forget to stress a certain point which is considered important. With an ideal textbook, of course, these points would be taken up carefully in turn, and there would be little chance to miss them; but few modern schools are able to find a textbook that exactly fits their course of study, unless the course of study is made out from the textbook, and certainly few would claim that such a course would fit local conditions.

In making or selecting the tests it is still easier to neglect some of the skills or items of knowledge if they have not been written down beforehand. If these specific aims contribute to the aims of the unit and subject as a whole, they should be included in the tests, especially those that are used primarily as teaching devices.

Section V. Suggested Activities and Exercises.

1. *Activities.* The meaning of activities as contrasted with exercises will be somewhat clarified by two quotations: "Exercises refer to those particular portions of subject matter to which the laws of learning and drill are particularly applicable; activities refer to the whole hearted, purposeful activity, or at least a large experience that will involve subject matter from several different fields or several parts of one field. Consequently exercises involving only the manipulation of facts

and ideas too often lack reality for the pupil. The activity is designed to bring reality back to the class room." (3:17) The activity intends to make the work clearer to the pupil and will naturally vary with local conditions. The activities suggested in plans submitted in Chapter V are all suggestions that the pupil be allowed to do something. In some cases there are imagined activities, as having the children imagine they form a bank and make deposits and the like; actual checks and deposit slips are used, however. In others the children actually do the thing, as when they measure the height of a tree or the distance across a stream by the use of congruent triangles or scale drawing. An activity method of teaching mathematics is not being proposed here, but it is suggested that the activity will go a long way in giving the pupil a thorough understanding of the unit. In many cases if the unit is introduced by means of some activity, the pupil will be much more likely to get the big fundamental meaning of the unit than if he is started immediately into a discussion of the terms or the solution of problems. Such a procedure is illustrated in Teaching Plan No. 1 included in Chapter V, in which it is suggested that the unit on insurance may be introduced by having the pupils imagine their parents have agreed to form a mutual insurance company. (2:39)

The activity may also be used to advantage throughout the unit to add interest and variety to the class discussion as well as to give the fundamental meaning to various parts of the unit as they arise. It will also help to make the teaching unit fit local conditions and will aid the pupils in seeing the application of the subject matter to practical affairs. For example, if the children actually measure angles out of doors and compute inaccessible distances, their interest is certainly heightened in the making and use of such measurements, and it seems reasonable to believe that the desired concepts are more easily acquired. Care will be necessary, however, to prevent such activities from developing into meaningless play. More specific suggestions as to the use of activities are included in each teaching plan.

2. *Exercises* may be thought of in terms of the quotation on page 32 as "those particular portions or subject matter to which the laws of learning and drill are particularly applica-

ble.'' In the field of mathematics then they will take the form of exercises assigned for the purpose of giving certain skills or abilities. These may be manipulative skills, or they may be abilities to make certain kinds of interpretations before or after these manipulations are completed. Probably all would agree that the ability to make interpretations is by far the most valuable of the things learned in school. This ability is dependent, however, to a large extent on the ability to manipulate processes and the results obtained by such manipulations. (53:241-251) For example, the ability to interpret angles in general is developed only after making many measurements and drawings of angles, triangles, and other figures.

The majority of the materials that constitute the content of the usual textbook in mathematics may be classed as exercises. Most of the exercises for a unit in mathematics will be taken from a textbook in the hands of the pupil or assigned from other books which the teacher may have available. There will be, in the main, three groups of these problems. First, there will be the exercises and problems that are intended for the entire group. They should be so selected as to give the pupils the required skills and abilities listed in the aims of the unit. If a good textbook is in use, these will be listed directly from the book. For many reasons, however, it may be desirable to include others in this list; the textbook material may be too meager, it may not fit local conditions, or it may be too hard for the pupils. Enough exercises should be included in this list to give most of the pupils the desired skills and abilities.

Second, there will be problems and exercises which are intended for the members of the group who are slower than the average in acquiring the skills and knowledge of the unit. If the teaching is to be efficient, this group must be given some sort of remedial work. Therefore, for extra drill purposes, the plan should include some more exercises and problems of about the same range of difficulty as those mentioned above. They may usually be had from other textbooks or from work or drill books.

Third, there will be problems and exercises intended for the small group of pupils at the upper end of the scale who

will need some individual attention. This group has been taken care of to some extent in the suggestion regarding activities above, but exercises and problems should be available. These exercises must be of quite a different character from those listed in the other two groups. They should not only be harder, but they should call for more knowledge about the unit if they are to hold the attention of this group of pupils. For this reason it seems desirable, if possible, that these exercises and problems be taken from books which may be placed in the hands of the pupils in order that they may be able to read the explanations there. If the problems are merely put in their hands, they will have to depend on the teacher, or someone else, to help them surmount many comparatively easy difficulties, which they could readily handle if they had access to the explanation in the junior high school textbooks. For example, in a unit on insurance, the pupils might be given some work dealing with automobile insurance, where insurance is allowed to a certain per cent of the value of the car which is a certain per cent of the factory value of the car when new. When this is carefully explained in a textbook, few, if any, of this group would have trouble with it. If only the problem is stated, however, some would certainly have trouble with it. As with the activities discussed above, these problems and exercises should be carefully graded in order that the pupils may advance regularly to as high a standard as possible within the time at their disposal.

Section VI. Teacher Procedure and Means. This section is made up of three parts: 1. The use of the teaching plan during the teaching of the unit; 2. the detailed procedure in teaching the unit; 3. the use of supplementary drill exercises. The content of each of these parts will be discussed briefly; the actual contents for six illustrative plans appear in Chapter V. 1. In the actual plans under The Use of the Teaching Plan During Teaching, seven parts of the plan are discussed showing why they are important and how they may be used in teaching. These seven things are subject matter, outline, pupil inventory, aims, activities and exercises, correlations, bibliography, and the testing program. As these are all discussed in this chapter and illustrated fully in the Chapter V, they are only listed here. In the plans they are treated from the standpoint of the par-

ticular unit. Much of this is in the form of suggestions which should be used by the teacher as class room conditions vary.

2. The detailed procedure in teaching the unit is designed to give the teacher, first, a method of introducing the unit and, second, a method or methods of teaching each part of it. This is an attempt to anticipate the teacher's difficulties and make suggestions as to how they may be avoided and what remedies to apply as a corrective in case they are encountered. This is also illustrated very fully in the plans submitted and hence is not discussed here.

3. The use of the supplementary drill exercises and problems has also been discussed above. The discussion included in each teaching plan is from the standpoint of the use of the supplementary drill exercises and problems for remedial purposes and to care for individual differences.

This entire discussion on teaching procedure is included, not because this method of procedure is considered perfect at all, but because it is considered valuable for the poorly trained or inexperienced teacher to have a definite procedure for using the plan, with which he may not be very familiar, and for teaching subject matter with which he is also not familiar. From the standpoint of the more experienced teacher it gives a method of presenting the material and should at least lead him to scrutinize his own methods more closely, even if he does not make any changes. Only the best have kept all these things in mind, probably, and perhaps even these will be stimulated to add important things that have been omitted.

Section VII. Correlations. There are two sorts of correlations considered in the teaching plan of a unit. One is the correlations possible with other subjects which the pupil may be studying. The other is the correlations or connections within the unit and within the subject, as mathematics. Correlations with life outside of school should be stressed in most of the activities of the class. By correlations with other subjects we mean any real connection between the two that will be of interest and of value to the pupil. For example, in the study of a unit on triangles in the seventh grade the teacher may well bring in the connection of angles and triangles with map making

in geography. The school ground could be mapped with crude instruments, and probably the class would develop much more interest in both subjects. Many more such correlations are found in the plans included in Chapter V. It is important that any correlation that is used be the result of a real connection and not of a forced one, merely for the sake of having a correlation.

The correlation within the unit and the subject is also very important. The pupil must get the interconnections within the unit and between the units, if the course in mathematics or in any other subject is to be a unified whole in his mind. This is not easy to do, and it requires constant vigilance on the part of the teacher. The teacher should note every topic that is specifically connected with some phase of the work already studied and call it to the attention of the pupils.

Section VIII. The bibliography will need little discussion. It consists of references for both the teacher and the pupil. It should be extensive enough to allow the teacher to make substitutions if he does not have all the books. The teacher's part of the bibliography should deal with both subject matter and methods of teaching the unit. The part dealing with subject matter should be somewhat in advance of that to be studied by the pupils in order to give him a wider outlook on the unit.

The lists of references given in the plans included in Chapter V are by no means exhaustive. Other books might deserve a place. It seemed undesirable to make the list very long in plans which were to be used in the classroom. The lists of references were chosen with two things in mind: (1) their value for the particular unit (in the writer's judgment) and (2) their availability for the teacher and pupil, for example, in the Tulsa Schools certain supplementary textbooks were supplied to every mathematics classroom. These were referred to when possible in the plans that were used experimentally in that school system.

Section IX. The testing program should be one of the most important parts of the teaching plan of a unit. It should consist of three types of tests: first, the pre-test which has been

discussed above under Section III; second, the tests to be used as teaching devices throughout the unit. These are usually short tests and should be made to cover all or most of the items of knowledge and skills which are listed under Aims in Section IV. Since these tests enable the teacher to discover the weaknesses of the pupils, practice to remedy these faults may be given to the individual pupil or to the class as a whole. The tests may of course be used as achievement tests for the part of the unit which they cover. Third, there should be the final test or tests, which may be given as a purely achievement test, and the marks may be recorded as the pupils' marks for the unit. These may also be used as a teaching device. If the pupils fall so low as to be considered failures for the unit, some of their specific weaknesses may be determined from these tests.

These tests may take any form desired but should be made as objective as possible. Much time will be saved in giving the tests and in marking the papers if this is the case. In the plans submitted with this study the items of knowledge are tested by asking questions which may be answered by one word, or for which a blank is left to be filled by one or two words. Other forms of objective tests may be used, however.

While the tests may vary considerably in form and from one unit to another, there are certain characteristics which all should possess: 1. They should test for all or most of the measurable aims of the unit. The items tested for determine to a large extent the items taught. If only the mastery of manipulative skills is being tested, then the mastery of these skills becomes "the major objective operating in the classroom." 2. Tests should be given both before and after the unit is taught. Otherwise there will be no control over individual differences, and one will be teaching all of the pupils things that some of them may already know. Also there will be no way of judging the efficiency of the instruction. 3. The tests should be capable of being used as a teaching device as discussed above. 4. The tests should be so constructed that they may be marked quickly and impersonally. It may be possible to find these principles involved in a test book or test series; if so, the teacher would not have to make them out for the unit. The stress placed on

objectivity here and elsewhere should not make the teacher lose sight of or omit the gauging of attitudes, appreciations, and the like. Though these cannot be measured objectively, they are very important.

The advantages or justification of including a testing program in a teaching unit plan should probably be discussed only briefly. First, if the tests are not made out or selected beforehand, many times the teacher will pass over a part of the unit and not check on the pupils' mastery at all. This will mean that, at least for a part of the class, he is assuming that the pupils have certain items of knowledge and skills which may have been very poorly learned or not learned at all. Second, many teachers are unable to make a test of this sort, either because of too full a schedule or lack of ability or experience; but if they had the tests made out, they could make very efficient use of them. Third, their use as a teaching device has already been discussed. Detailed explanation as to how and where each test should be used and the use made of the results with suggestions as to what remedial work may be used should be included as a part of Section VI, Teacher Procedure and Means, of the teaching plan.

Planning by Experienced Teachers

The value of any teaching procedure or method is determined to a certain extent by the use made of it by successful experienced teachers. In order to determine how closely the technique used by experienced teachers corresponds with the teaching plan of a unit, questionnaires were sent to the teachers of eighteen junior high schools in sixteen different states. The names of the organized junior high schools in the United States were obtained from the Department of Education in Washington. Letters were then sent to the principals of seventy of these junior high schools to determine if they were willing to request their teachers to answer the questionnaires. The return card sent with these letters contained four questions. The return card and questionnaire are reproduced below.

QUESTIONAIRE

School: .....Subjeet or Grade.....

Name: .....No. Yrs. Experience.....

The questions have to do with teaching practiee among experienced teachers in regard to teaehing units or topies. In general assume that you are about to teach some unit or topie with whieh you are thoroughly familiar and have taught several times. We shall want to know four things about eaeh item (1 to 9) mentioned below. Before beginning the unit,

1. Do you have these items definitely in mind?
2. Do you write them down?
3. Do you merely outline them?
4. Do you make no use of them? (Please cheek)

	In mind	Writ-ten	Out-line	Make no use
1. Aims a. General				
b. Specific				
2. Pupil inventory a. Intelligence quotient				
b. Social background				
c. Previous knowledge of the un t				
3. Activities to motivate work				
4. Exercises for extra work a. For slow group				
b. For best group				
5. Outline of unit				
6. Teaching procedure for entire unit				
7. References a. For pupil				
b. For teaehers				
8. Correlation with other subjeets				
9. Testing program for whole un't				

1. Would an inexperieneed teaeher do better work in your opinion if he had the above items written out for the units he teaehes? Yes..... No.....
2. Would the experieneed teacher do better work in the majority of eases if he had the above items written out for eaeh unit he teaehes? Yes..... No.....

Remarks:

Return Card

To be filled out by junior high school principals
School: .....
Address: .....
Principal: .....
Are you willing to ask your teachers to answer the questionnaires? Yes..... No.....
Do you think the results valuable? Yes..... No.....
Does your supervisory program include any form of written lesson plan? Yes..... No.....
Number of experienced teachers.....

Replies From Principals

Of the seventy principals to whom the letters were sent, twenty-five returned the enclosed card. Of this number nineteen agreed to request their teachers to fill out the questionnaire. In one case the teachers voted "No," and five principals said they were not willing to request their teachers to fill out the blank. A large majority of the principals thought the study would be of value, nineteen voting "Yes," one "Doubtful," and five failing to answer the question. In eleven of the cases the supervisory program included some form of the written lesson plan, in nine it did not, and the same five principals failed to answer the question.

Replies From the Teachers

Usable replies were received from one hundred and seventy-seven teachers in ten different states. Questionnaires were sent to the principals of eighteen schools. Thirteen principals returned the questionnaires, one school closed before they arrived, in one case the questionnaires were lost during the principal's absence and three principals failed to return them. The experience of the teachers varied from one to forty-two years, with a mean of 10.3.

Use Made of the Items of the Teaching Plan

Table I shows the way experienced teachers use each item of the teaching plan, the total who make use of it, and the number who make no use of the items.

TABLE I

How experienced teachers use the items of the teaching plan.

	In mind	Written	Outline	Total	No use
1. Aims					
a. Specific	87	52	32	171	6
b. General	58	77	28	163	14
2. Pupil inventory					
a. Intelligence	97	51	10	158	19
b. Social background	142	11	7	160	17
c. Previous knowledge	121	26	20	167	10
3. Activities to motivate work	68	60	39	167	10
4. Exercises					
a. For slow group	28	89	36	153	24
b. For best group	34	90	36	160	17
5. Outline of unit	13	108	51	172	5
6. Teaching procedure	61	54	58	173	4
7. References					
a. For pupils	33	101	29	163	14
b. For teachers	56	71	30	157	20
8. Correlations	126	16	16	158	19
9. Testing program for whole unit	27	122	17	166	11

An examination of the table shows that a large majority of the teachers made some use of the items of the teaching plan. It will be noticed that a large number of teachers kept the items "in mind." This is perhaps all that is necessary in some cases, and is probably all that can be expected of the busy teacher. These results would destroy to some extent the argument for the use of the teaching plan for experienced teachers. Table II, however, shows that the majority of the teachers are of the opinion that experienced teachers would be able to do better teaching if they had these items written out before the unit had begun. Table I shows that a much smaller number actually write the items out. Table II shows, also, that an overwhelming majority think that the inexperienced teacher could do better work if he had a written plan.

TABLE II

Opinions of teachers regarding the value of having items written out.

	YES	NO
Of value to experienced teachers	116	46
Of value to inexperieneed teachers	162	8

This study shows that the items of the teaching plan are actually used by a large majority of experienced teachers, and it is reasonable to assume that at least these items are of importance in planning the teaching of any unit. The opinions of the teachers in regard to the value of having the items written out would seem to indieate that the teaeher should have a written plan in his hands if he is going to do the best possible teaching. It is quite obvious to one who has attempted to work up such material that few, if any, teachers will have the time, or should take the time neeessary to work up such teaching plans for all the units they will have to teach during a year. It is possible that the work should be done by experts and be turned over to the teachers eomplete. However, the individual teacher may advantageously work out a few plans each year and thus in time have his work well in hand, though the plans should be ehanged eonstantly to meet new eonditions.

CHAPTER III

Experimental Results of the Use of the Teaching Plan

This chapter includes the statistieal results and interpretations of the use of four of the teaching plans in the public schools of Oklahoma. Three of these plans, Plan II on *How to Use Equations,* Plan III on *Practical Indirect Measurements,* and Plan IV on *Square Root, Radicals and Exponents,* were used by teaehers in the Tulsa Publie Sehools, Tulsa, Oklahoma; and the fourth, Plan I on *Insurance,* was used in fourteen small town and rural schools in the Southeastern Distriet of Oklahoma.

The experimental and eontrol group method of eomparison was used. With one group, the experimental group, the teaeher or teachers used a teaching plan, (submitted later in the dissertation) to aid in teaching the unit. With the other group, the control group, the teaeher or teaehers used no sueh plan but taught it as they ordinarily would have done. In many eases neither pupils nor teaehers of the eontrol group knew that an experiment was being earried on. In every instanee the control group and the experimental group were taught identical subjeet matter. These units were taken direetly from units carefully outlined in the eourse of study with time limits and eorresponding page numbers in the textbooks given. These outlines were in the hands of all teachers at the beginning of the semester. The subject matter of the units was thus available to all teaehers alike.

The teachers of the experimental groups did not teaeh any pupils of the eontrol group. This deeision was based on the assumption that the teaeher in studying the teaching plan would aequire items of knowledge about the unit and its organization that would make it impossible to teaeh the unit as he would have taught it before. The teaehers for the experimental groups were ehosen by merely asking the prineipals

to suggest some teachers who might be willing to help in the experiment, and who were to teach these particular units. Some of these consented to do it and some did not.

In each case the groups of pupils were equated according to intelligence. Control groups in most of the instances used were rather large and consisted of children taught the same course of study in the same school system, and it was assumed that equating on the basis of intelligence would give equal groups on the basis of mathematical ability. (19:57-58) The scores on the unit achievement tests, given to both groups at the close of the unit, were used as a measure of their success on the unit.

Plan I, on *Insurance*, was used during the second semester of the year 1929-30. As stated above, fourteen different schools participated in the experiment, a part of them using the plan and a part of them serving as a control group. This plan was built on the chapter in the adopted textbook (10) which was used as a course of study for the schools.

The teaching plan of this unit was sent to the teachers of the pupils in the experimental group before the unit was scheduled to begin. A sheet of written instructions was sent with the plan to help the teachers carry out the experiment. A list of questions on the plan was also sent to help them understand its meaning. Copies of the tests were included for the pupils also. The teachers taught the unit and returned the tests after the pupils had taken them. These tests included the Unit Achievement Test which was given at the end of the unit.

The teachers of the control group were asked to teach the unit as they ordinarily would and to give the Unit Achievement Test (Test III, Plan I) at the end of the unit. The teachers of both groups were asked to observe the time limit (three weeks) carefully in teaching the unit. Since the subject matter taught was identical and the time element, the same, the results of the tests should have favored neither group of pupils. All of the Unit Achievement Tests for both the experimental and the control groups were marked by the writer and the scores used as a measure of the achievement of the unit.

As the schools were in the smaller towns, no intelligence ratings of the pupils were available. To secure such a rating the Miller Mental Ability Test, Form A (20), was given to both groups. From the scores on these tests it was possible to figure the intelligence quotient scores (21:19-20) which were used in the statistical work below.

The teachers of these two groups were probably about equal in ability, but were made up of good and fair teachers. Not all of them were experienced teachers. Having a number of teachers of varying ability introduces a variable which it is almost impossible to control, since even teachers in a good school system are not of equal ability when it comes to teaching a class in such a way that the pupils will be able to pass a particular kind of test.

Results of the Use of Plan I

Table III gives the mean, median, and standard deviation of the scores of the two groups on intelligence and on the Unit Achievement Test. The intelligence ratings were the I Q scores mentioned above. These results show that the groups were

Table III

Comparison of the experimental and control groups for Plan I

	GROUP	NO. PUPILS	MEAN	MEDIAN	S. D.
I Q	Exper.	112	100.45±0.63	98.50±0.79	9.91±0.45
	Cont.	156	100.03±0.57	99.66±0.72	10.58±0.49
A T	Exper.	112	61.79±1.51	65.45±1.89	23.36±1.07
	Cont.	156	53.27±1.60	55.00±2.00	29.62±1.13

To be read: The mean I Q for the experimental group was 100.45 with a probable error of 0.63; the standard deviation of the I Q of the control group was 10.58±0.49; the median of the achievement test for the control group was 55.00±2.00.

about equal from the standpoint of intelligence, as measured by these tests. On the Unit Achievement Test, however, the mean of the experimental group was 8.52 higher and the median 10.45 higher than the mean and median of the control group. The standard deviation, which serves as a measure of the spread of the distribution, was 6 points lower for the control group. Thus the central tendency was lower, and the distribution was

more scattered for the control group than for the experimental group.

Table IV shows the actual differences of the means and medians and the probable errors, of the differences for the achievement test only. The means and medians of the intelligence scores were so nearly equal that the difference has no significance. The probable error of the difference of the means for the achievement test is 2.20. This means that if a large number of means shoud be computed, each based on the same number of cases and on data similar to these, it is probable that half of the differences of the means would lie between 8.52—2.20 and 8.52+2.20. Stated differently, the chances are equal — 50 in 100 — that the true difference lies between 6.32 and 10.72. It will be noted that three times the probable error of the difference (6.60) is less than the actual difference, while

Table IV

Actual differences of the means and medians and the probable errors of the differences for the two groups of Plan I

	Act. diff	P. E. diff	3 P. E. diff	4 P. E. diff
Mean A T	8.52	2.20	6.60	8.80
Median A T	10.45	2.75	8.25	11.00

To be read: The actual difference between the means of the achievement test = 8.52; the probable error of the difference of the medians of the achievement test = 2.75; four times the probable error of the difference of the means of the achievement test = 8.80, etc.

four times the probable error of the difference (8.80) is just over the actual difference (8.52). While this difference could hardly be ascribed to the fluctuations of sampling, (61:345) it should not be called highly significant. (25:227) Since the same thing is true in regard to the difference of the means of the two groups, it may be said that the difference in favor of the experimental group seems to be a fairly significant one, though not highly significant.

It was also possible to select two equal groups of pupils on the basis of intelligence, from the experimental and the control groups mentioned above. This was done in the following manner; the scores of each pupil on the intelligence test and

on the unit achievement test were placed on a separate card. Each group of cards was then arranged in the ascending order according to the I Q scores. This was done with no reference to the unit achievement test scores. Thus all of the 84s, all of the 85s, all of the 102s, and the like were placed together at random. This gave two piles of cards, the experimental group and the control group. The lowest card was then taken from one pile and matched exactly with a card from the other pile; if there was no exact match, it was discarded and the next

Table V
Comparison of the experimental and the control groups for Plan I, based on the equated scores

	Group	No. Pupils	Means	Medians	S. D.
A T	Exper.	88	62.39±1.65	66.32±2.07	22.93±1.17
	Cont.	88	53.52±1.97	55.00±2.53	28.10±1.43

lowest card matched, if possible. This procedure was repeated until all the cards had been either matched or discarded. This gave two groups of 88 each, exactly equated on the basis on intelligence, as measured by these tests. The mean I Q score of each of these groups was 99.83 which, it will be noted, is practically the same as the mean scores of the larger groups discussed above.

Table V gives the results of the unit achievement test for these two equated groups. Almost the same differences are

Table VI
Actual differences of the means and medians and the probable errors of the differences for the two equated groups of Plan I

	Act. diff.	P. E. $_{diff}$	3 P. E. $_{diff}$	4 P. E. $_{diff}$
Mean $_{A\ T}$	8.87	2.57	7.71	10.28
Median $_{A\ T}$	11.32	3.27	9.81	13.08

present as were noted in Table III above. Table VI shows the actual differences compared with the probable errors of the differences for both the mean and the median. Since the actual difference is greater than three times the probable error of the

difference but less than four times the probable error of the difference, we may say that the results are fairly significant, but not surely so.

. These two groups were also compared in another way. As has been stated above, the I Q scores were determined from Form A of the Miller Mental Ability Test. The formula for finding the I Q as given by Dr. Miller is

$$\frac{2S+80}{C\ A} = I\ Q,$$

where S is the raw score on the test and C A is the chronological age in months. The raw scores alone will give a measure of the pupils' ability which is considered by some to be more reliable for comparative purposes than the derived score. The raw scores for some of the pupils in the experimental group were lost; hence this comparison is made on the basis of an experimental group of 85 and a control group of 156. The results of this comparison are set out in Tables VII and VIII. The differences in the mean and median scores on the intelligence test are less in each case than three times the probable error of the difference. These differences can therefore be said to

Table VII.

Comparison of the experimental and control groups for Plan I, based on the raw scores of the intelligence test.

	Group	No. Pupils	Means	Medians	S. D.
I Q	Exper.	85	45.72±1.05	45.67±1.34	14.70±0.76
	Cont.	156	43.32±0.72	42.33±0.92	13.64±0.52
A T	Exper.	85	59.82±1.69	62.50±2.16	23.66±1.22
	Cont.	156	53.27±1.60	55.00±2.00	29.62±1.13

be of no significance. The differences for the mean and median scores on the achievement test were also small but were larger in proportion. For both the means and the medians the difference was greater than twice the probable error of the difference but less than three times the possible error of the difference. These differences should certainly not be considered highly significant and should probably not be considered significant at all. (61:245)

Table VIII

Actual differences of the means and medians and the probable errors of the differences for the two groups of Plan I, based on the raw scores of the intelligence test

	Act. diff.	P. E. diff	2 P. E. diff	3 P. E. diff	4 P. E. diff
Mean I Q	2.40	1.27	2.54	3.72	4.89
Median I Q	3.34	1.63	3.26	4.90	6.54
Mean A T	6.55	2.33	4.66	6.98	9.30
Median A T	7.50	2.59	5.90	8.85	11.80

These three ways of considering the data give approximately the same results. None of the three shows a highly significant difference between the means and medians of the achievement test scores. One would probably be justified in saying that the chances are about 20 to 1 that with similar data the actual differences would be within the limits noted above.

Plans II, III, and IV were used by various teachers in the junior high schools of Tulsa, Oklahoma, during the first semester of the school year 1930-31. These plans are based on the course of study in the Tulsa Public Schools, 1930, with the Schorling and Clark junior high school series as the adopted textbooks. (37) (38) 39) From one to four teachers participated in the use of each plan. Since the Tulsa schools were in the midst of reorganizing their course of study, the plans were made to fit their tentative units exactly; that is, teachers using the plans were teaching subject matter identical to that being taught in all the schools. This also had another advantage. The city course of study included certain tests, called inspection tests, which were given to all the pupils. These tests covered the units fairly well in most cases; hence it was possible to use them as Unit Achievement Tests. This had the added advantage of giving a large control group.

The teachers who agreed to do the experimental work were given the plan of the chosen unit some days before the unit was scheduled to begin. They were asked to read it carefully and ask questions about any point. The teacher and the prin-

cipal of the school were interviewed by the writer in each case. The general purpose of the experiment, the general idea of the teaching plan, and some of the details of the plan were discussed with each teacher. Some of the teachers were also interviewed while the unit was being taught. All of them were teachers of rather wide experience and training, and were teaching in an excellent school system in a city of about 150,000 population. They were told to teach the unit to the best of their ability and to use the plan if it were possible to make it fit into their work. The time to be spent on each unit was rigidly set by the assistant superintendent of schools in charge of the curriculum.

The pupils of these teachers formed the experimental group referred to above. Larger groups of students chosen at random from the same grade made up the control group. The average achievement of this control group was probably nearer to the standard achievement of the grade than any particular group chosen from some particular school would have been. The teachers of these larger groups knew nothing about the experiment.

Results of the Use of Plan II

The results of the use of Plan II, on *How to Use Equations,* will be found in Tables IX and X. The I Q scores were taken from the records of the Tulsa Public Schools and were deter-

Table IX

Comparison of the experimental and control groups for Plan II

	Group	No. Pupils	Mean	Median	S. D.
I Q	Exper.	89	113.51±0.84	112.68±1.09	12.68±0.61
	Cont.	360	111.34±0.55	111.41±0.71	15.87±0.40
A T	Exper.	89	22.33±0.42	23.90±0.53	5.97±0.30
	Cont.	360	20.30±0.23	20.89±0.29	6.55±0.16

mined by the use of Form A and Form B of the Terman Group Test for Mental Ability. (51) The raw scores of these tests were not included in the record. Reference to the Unit

Achievement Test of Plan II will show that the highest possible score was 30. In Table X the actual differences of the means and medians and the probable errors of the differences of the means and medians are given for both intelligence and achievement. Since the actual differences of the mean and median scores for the intelligence of the two groups are considerably less than three times the probable errors of the differences, one is justified in assuming that there is no significant difference in the intelligence of the two groups. The actual difference

Table X.
Actual differences of the means and medians and the probable errors of the differences for the two groups for Plan II

	Act. diff.	P. E. diff	3 P. E. diff	4 P. E. diff
Mean I Q	2.17	1.18	3.54	4.72
Median I Q	1.27	1.29	3.87	5.16
Mean A T	2.03	0.48	1.43	1.90
Median A T	3.01	0.61	1.83	2.44

in the means and medians for the achievement tests are greater, however, than four times the probable error of the differences. This indicates that these are significant differences. (32:231)

Results From the Use of Plan III

The results of the use of Plan III, on *Practical Indirect Measurement,* are presented in Tables XI and XII. The pupils of the experimental group were much lower in I Q than any other group of Tulsa pupils who took part in the experiments. This may have been due to the fact that the school was in one of the poorer residential sections of the city. The difference in the mean I Q scores of this group and the control group

Table XI
Comparison of the experimental and control groups for Plan III

	Group	No. Pupils	Mean	Median	S. D.
A T	Exper.	35	5.13±0.11	5.30±0.27	1.87±0.15
	Cont.	35	3.59±0.25	3.50±0.32	2.22±0.18

used was about 10 points. For this reason it was necessary to find a group equal to this group in I Q and use this as a control group. This was done in much the same way as the method described above in the discussion of Plan I. The scores for I Q and Achievement for each pupil of the experimental group were placed on separate cards. Like scores of a larger group

Table XII

Actual differences of the means and medians and the probable errors of the differences for the two groups of Plan III

	Act. diff.	P. E. diff	3 P. E. diff	4 P. E. diff
Mean A T	1.54	0.32	0.97	1.29
Median A T	1.80	0.42	1.25	1.66

of pupils from a school in a similar district were also placed on cards. Then each of the cards of the experimental group was matched, according to intelligence, with one from the larger group. In a few cases where a card could not be exactly matched one that differed by one point was chosen. These were so chosen that the errors balanced each other. The mean I Q for each of the groups so chosen was 101.74±1.35 and the median was 100.50±1.73. Table XII shows that the actual differences for both the means and the medians were more than four times the probable errors of these differences. This would indicate that the difference in achievement of the two groups is certainly significant. Care must be taken, however, not to base important conclusions on the results found from the use of so small a number of cases.

Table XIII

Comparison of the experimental and control groups for Plan IV

	Group	No. pupils	Mean	Median	S. D.
I Q	Exper.	38	107.50±1.48	107.86±1.85	13.51±1.05
	Cont.	117	107.59±0.99	107.02±1.24	15.81±0.70
A T	Exper.	38	22.74±1.00	25.00±1.26	9.29±0.71
	Cont.	117	17.80±0.42	17.13±0.53	6.79±0.31

Results of the Use of Plan IV

The results of the use of Plan IV, on *Square root, Radicals, and Exponents* are found in Tables XIII and XIV. It will be noted that the two tests at the end of this plan are on different parts of the unit with no duplication in the questions asked. For this reason the combined scores of the two tests was taken as the measure of the achievement on the unit. The highest possible score was 35. The I Q scores were taken from the records of the Tulsa Public Schools.

Table XIV

Actual differences of the means and medians and the probable errors of the differences for the two groups for Plan IV

	Act. diff.	P. E. $_{diff}$	4 P. E. $_{diff}$
Mean $_{A\ T}$	4.93	1.09	4.36
Median $_{A\ T}$	7.87	1.37	5.48

In Table XIV the actual differences of the means and medians and the probable errors of the differences of the means and medians are given for achievement scores only. The differences of the means and medians of the I Q scores are so small that they are not significant. Since the actual differences of both the means and medians are each greater than four times the probable errors of these differences, the differences are certainly significant. It should be observed, however, that the experimental group was rather small in this case also, and too much significance should not be attached to the results.

Conclusions

1. The experimental groups made the best scores on the achievement test for each of the four units used.

2. In three cases these differences of the means and medians were greater than four times the probable error of the difference; this indicates that they were highly significant differences.

3. In the other case (Plan I) the differences of the means and medians were greater than three times the probable error

of the difference; this is an indication that while the difference could not be attributed to the fluctuations of sampling it may not be interpreted as highly significant.

4. These results seem to indicate that the use of the teaching plan of a unit in the actual classroom situation would result in better learning of the type necessary to succeed on the tests given.

5. Care must be taken to consider other factors that are beyond control in an experiment of this sort. Some of these are: the varying ability of teachers to teach children in such a way that they will be able to succeed on a certain type of test; the general attitude of the pupils often determined by administrative policies over which the teacher has no control; the previous teachers which the pupil may have had, and the like.

CHAPTER IV

The Use of the Teaching Plan as a Part of a Teacher Training Program

The statements made in this chapter are based on the personal experience of the writer and some of his colleagues in the Russell Training School of the Southeastern State Teachers College at Durant, Oklahoma, where the teaching plan of a unit of work has been used in all grades through the junior high school since 1927 in connection with the teacher training program of the school. This chapter does not contain any objective evidence as to the usability of the teaching plan in a teacher training program. It is largely a description of a group project evolved and used by the teaching staff of the above mentioned training school, together with a summary of the expressed opinions of administrators, teachers, and student teachers.

The general method of procedure as it is actually carried out will be described first, and later a more theoretical procedure will be proposed for a hypothetical case in which certain difficulties existing in this particular school would not be present. The director of the training school has charge of a course in general education which all of the student teachers have to take. For one phase of this course the student is given a general and necessarily brief introduction to the teaching plan of a unit in some elementary subject. The student is usually asked to make out a plan on some brief teaching unit in the elementary field in order to give him a first general idea of the meaning of a teaching unit and the meaning of the teaching plan.

This work is given with the view of building on it when the student takes, usually in the following term, the first course in observation and practice teaching. In this course he is under the direction of the critic teacher; but since the director has general supervision of the work, there is a definite unity running through the whole plan. Each student is required to take three nine-week courses in observation and practice teaching;

the first of these is called Observation of Teaching, the second Participation in Teaching, and the third Practice Teaching. At the beginning the student chooses the group in which he wishes to work. There are four such groups: primary, consisting of grades one and two; elementary, consisting of grades three and four; intermediate, consisting of grades five and six; and the junior high school, consisting of grades seven, eight, and nine. The student teachers for whom this course is intended are called the two-year normal school group and are granted life certificates at the end of two years of training and, as such, are not permitted to teach in the high schools; hence this training does not cover that field. Student teachers who choose to take their practice teaching in any one of the first three groups mentioned above observe equal lengths of time with the two teachers of the grades. Those who choose to do their work in the junior high school divide their time equally among the teachers of mathematics, English, and social science. This is done to give the student as wide a range as possible and still leave each teacher enough time to introduce him carefully to the type of work in the grade or subject. In the junior high school the student observes in each of the grades, but a different subject in each, as, for example, English in the seventh grade, mathematics in the eighth, and social science in the ninth. This gives him the advantage of seeing the whole range of pupils as well as these three important subjects. In the first course, Observation of Teaching, he has a chance to see the teaching plan in use. The critic teacher tries to arrange his work so that the student may observe the beginning of a unit. Many times there is not time to finish the unit before other phases of the observation work must be taken up, or the student has to begin observation with another teacher. Upon the entrance of the observers, the critic again explains the teaching unit and the teaching plan with especial reference to the way it is used in this particular grade and subject. The student then observes the teacher who is teaching with a written plan, reads certain references, and is required to submit reports on his reading and observation. Other things are, of course, given emphasis during this period, as types of daily lessons, classroom management, and the like. During this period the student's work is some-

what scattered and to some extent disorganized since he has to change teachers and subjects several times, but he is required to observe closely and read rather extensively and reduce to writing many of his opinions about crucial points.

For the next period his work becomes more specialized. He is required to choose, in conference with the director of the training school and the critic teacher, the subject and grade in which he wishes to work for the following two terms or eighteen weeks. For the first nine weeks of this period he is registered for what is called a participation course, for which he gets two semester hours of college credit. This is made up largely of individual work under the direction of the critic teacher, with a gradual introduction into the management of the class toward the last of the term. The student is expected to observe the class he will be teaching the next term long enough at least to learn the names of the pupils and something about their class standing. Usually he observes most of the time but may be excused to do reading or other types of work. In general there are four aims in view in giving the course:

1. To vitalize courses in education already taken by the student, such as Educational Psychology, Child Psychology, Adolescent Psychology, and the like, by letting the students see pupils in actual situations in the class room while he is rereading some of the material.
2. To give the student an opportunity to master the subject matter which he will be required to teach the following term.
3. To give the student an opportunity, under the direction of the critic teacher, to get general and specific ideas about the methods of teaching the grade and subject chosen.
4. To give the students time to make teaching plans for the units which the class will cover the following term.

The first three aims are for the purpose of accomplishing the fourth. They are to give the student background and material to enable him to write the teaching plans for his next term's work. There is, of course, danger that this reading may be poorly done or neglected altogether; a certain number of

written reports are required, but the problem is still not solved for the student who wants to shirk as much as possible.

After the student's reading is under way and he has gained some background in methods of teaching the subject, he begins on the plan of one of the units he is to teach. Experience seems to indicate that the student now, for the first time, begins really to understand the teaching plan of a unit. Work of this kind usually makes the student realize the value of his reading, especially in the field of methods. He may now read for a definite purpose in order to find ways of teaching definite topics and definite groups of children. The student works out part of the teaching plan, which then at his regular conference with the critic teacher is criticized, and suggestions are made as to revisions. In most cases, the student gets the first draft wrong; either the material is out of place within the plan or entirely irrelevant. It is placed correctly by the critic teacher, who explains his reasons for the action. Although the student's work has been wrong, he gets a clear idea of the meanings of the parts of the plan involved. This process is continued in the conferences until the teaching plan is completed by the student. Then it is criticized as a whole by the critic teacher.

If at all possible the student is allowed to teach a short unit to the class as part of the gradual introduction into a position of responsibility. This makes the student see, as he can in no other way apparently, the weak spots in his plans. In some cases another of these short units is taught later in the term.

The student continues to read and work on the plans until the end of the nine-week term. The plan is to have him finish the teaching unit plans for all the units he will be required to teach during the following term. The construction of the plans is not perfectly done, and usually the student could do much better on the plan of the second unit after he has taught most or all of the first one, but he will not have time to make such a plan during the time he is teaching. He should be able to make some revisions in the light of his experience.

There is a definite feeling on the part of the critic teachers that students do a much better type of teaching during practice

work when they have had the work on planning and participation mentioned above than was done when it was not given. Since it is a new requirement and some students cannot take the course, because of conflicts or lack of time, the critic teachers have a better basis of comparing the value of the two methods of preparation than they would have if the new course had been added for all and they were depending on memory alone.

During the last nine weeks the student tries out his plans in the class room. He has an opportunity to see how his ideas work out and to judge the value of the various parts of the plan. The critic, of course, gives any assistance he can. It is a general student teaching situation in which the student has thought through and carefully planned each of the units he expects to teach.

Opinions of Teachers and Students Who Have Used This Plan of Procedure

As stated above the teaching unit plan has been used in the teacher training program of the teachers college at Durant for the past three years and in all grades through the ninth. Ten teachers and directors, not counting the writer, have had definite experience with this line of attack. Their opinions in regard to three important phases of the program are summarized below. In the group two held Ph. D. degrees, four M. A., three B. A., and one had no degree. The range of teaching experience was from eight years to twenty-five years. Most of the following data were summarized from answers to three questions quoted below:

1. What is the value of the teaching unit plan in a teacher training program?
2. What is the value of the participation course to the student when he does his practice teaching? Compare with students who do not take it.
3. In your opinion and according to your experience what is the best way or ways to give the student teacher a working idea of the teaching plan of a unit of work?

Summary of Expressed Opinion

I. The value of the teaching plan in a teacher training program.

All of the ten teachers mentioned above rated the value of the teaching plan of a unit high as a procedure in training teachers. Two quotations as to its value might be in place.

From the former director of the school, now director of Teacher Training for the State of Oklahoma:

"I have a very high estimate of the value of the teaching unit as a particular procedure in training teachers. It did more to raise all our teacher training efforts to a functional level than anything we tried — and we tried many things — . This program is the only one I have ever seen tried that was effective in training teachers to be purposeful."

The other director feels that

"teachers trained under the present plan are much better qualified to do a satisfactory piece of work . . . It affords the student teacher an opportunity to apply every item of educational importance which he has learned in his entire course in the teachers college."

The teachers listed a number of values or advantages of using the teaching unit plan in a teacher training program. They are summarized below:

1. The work of organizing a unit of instruction brings into operation the student teacher's knowledge of psychology, tests and measurements, his aims and objectives, activities and procedure, material of instruction and desirable outcomes.
2. The teaching unit plan requires the critic teacher to organize the subject matter from three standpoints, student teacher equipment, subject matter requirements, and child interest and comprehension.
3. It compels the student teacher to comprehend subject matter in terms of educational values.
4. It makes the child the center of all teaching activity.
5. It shows the interrelation of various activities which the child experiences in and out of school.
6. It provides for this interrelationship by using these experiences as a basis for instruction.
7. It provides for connected thinking on the part of the teacher and pupils.

8. It insures careful preparation.
9. It insures enrichment.
10. It enables rural teachers to handle more than one grade at the same time.
11. It provides for self-directed activity on part of the children.
12. It makes homelike informality possible at school.
13. It makes teacher pupil relationship more natural, more social.
14. It provides for individual differences.
15. It presents a broader view of the whole teaching activity to the student in training.
16. It leads the student teacher to evaluate teaching material, not as an end in itself, but as something to be used in directing the child's activities.
17. It shows the student teacher the component parts of the teaching activity put together in a related whole.

II. The value of the participation course.

In dealing with the second question on the value of the participation course as a preparation for student teaching the answers were also in the affirmative by all the teachers. Many students who were taking the course were also asked for their opinions.

Summary of the Values of the Course in Participation

1. A participation course guarantees that the student teachers think through the subject matter from every possible angle.
2. It causes subject matter to cease to be so many facts to be organized, comprehended, and in some instances memorized, and to become human experiences to be revitalized through proper presentation.
3. It prepares the student teacher to start his practice teaching.
4. It gives the inexperienced teacher the opportunity
   a. To adjust himself to the child's point of view.
   b. To study child nature and ways of meeting behavior problems.
   c. To study by observation or participation various phases of the teaching situation:
      (1) types of activities
      (2) kinds of procedures
      (3) means of motivation

(4) ways of providing for individual differences
(5) classroom routine and record keeping.

d. To study himself and study teaching techniques that best fit his temperament—those that he can use to the best advantage.
e. To obtain directed professional reading.
f. To collect materials useful in teaching.
g. To study plans and to write teaching plans on various kinds of units.

III. Summary of suggested methods of giving the student teacher a working idea of the teaching unit plan.

This is a summary of methods suggested by the group of critic teachers and administrators. The list is not intended to be exhaustive.

1. Make the unit idea prominent in every course in the teachers college.
2. Make it an important topic in the course in general methods.
3. Discuss its psychological aspects in the course in educational psychology.
4. Give an apprentice period just preceding the practice assignment in order to master subject matter and to break it into suitable units.
5. Observe critic teachers in teaching a well prepared teaching plan.
6. Prepare such a teaching plan under the direction of the critic teacher.
7. Give time for teaching short units and discuss flaws in conference.
8. Rewrite the teaching plan and go on to the preparation of others.

A Suggested Use of the Teaching Unit Plan in a Teacher Training Program

No attempt is made here to describe a complete teacher training program; such a task lies beyond the scope of this study. Suggestions made here would probably be hard to follow in detail in most practical situations; but if the suggestions will aid teachers or schools to adapt the work they offer to the needs of student teachers, the effort expended will be amply repaid. No effort has been made to justify the use of the teaching unit in this capacity, however. This is given as a modification of the program in use in Durant and seems to the

writer to present fewer undesirable features from the standpoint of the methods and critic teachers.

1. *Preliminary introduction of the teaching unit plan during some course in general education.* Ideally the student's first introduction to the teaching plan of a unit of work should take place sometime before he is called upon to use it. It could probably be introduced best during a discussion of the units of work or teaching units when the general topic of subject matter organization is dealt with. Units of work or teaching units seem to be a topic important enough to find a place in the general methods course. The teaching plan could be made to follow as a matter of course.

It could also be mentioned to advantage in the parts of the course dealing with the study of the pupil, aims, procedure, and the like. The teachers of these courses could probably decide more easily just where it should be introduced and with what emphasis.

2. *Introduction of the teaching unit plan as a part of the methods or professionalized subject matter courses.* The amount of time spent on the teaching plan in one of these courses would depend on the time allowed for the course. Many such courses are much too short. It is reasonable to assume that in the training of any teacher, and more especially of the junior high school teacher, either a methods course or a professionalized subject matter course should be given in the subjects he expects to teach. In either type of course not all the time should be spent on general principles. Some time should be given to the discussion of the subject matter to be taught and to a discussion of how to teach it. These courses should be given by an expert in the subject matter field.

If the subject matter of junior high school mathematics, for example, is organized into teaching units, a teaching plan of a unit could be used in the discussion of specific methods of teaching it. If the teaching plan is to be made a serious part of the course, however, the matter must go deeper. A careful study should be made of the teaching plan as a whole and of its parts separately. This could be done best, probably,

through sample plans in the hands of the students. This should be followed by laboratory exercises during which the students work on plans of their own, dealing with important topics of the junior high school mathematics course. This will give them a much more definite idea of the meaning of the teaching plan and of all its parts, and also a more vital understanding of the methods of teaching the subject matter included.

Some use could be made of the teaching plan while one is studying methods of procedure in teaching particular parts of the subject matter. For example, in dealing with angle measurement, the student could be shown its use in the particular part of a unit plan in which the pupil proves that the sum of the angles of a triangle is equal to 180°. This would give the student a deeper appreciation of the value of these methods, the use of which is usually so remote that the interest he has in them is largely academic.

3. *Use of the teaching unit plan during the observation period in the training school.* If the student teacher has had the training outlined above, he will come to the training school prepared to take advantage of the teaching plan used by the teacher. This plan, if studied carefully by the student, will show him many things about the teaching he is to observe. It will reveal to him the type of pupil in the class, the teacher's aims and expected outcomes, the subject matter to be taught, the activities which may be used, the methods of procedure, the tests to be used, and the like. The critic teacher should make some sort of check to see that the observer has actually perceived these things.

If the student teacher knows what to look for, he is much more likely to see it and to see how the teaching was done, and how he can later accomplish the same results. It will be the task of the critic teacher to devise some means to aid further the student in doing this and also to devise some means of checking to see that he is actually able to do it before the observation period is over.

4. *The use of the teaching plan during participation and student teaching.* These courses are discussed together because

it is felt that the courses should be combined in order that there will be alternate periods of student teaching. No student teacher should be expected to do all his planning before any teaching has been done. Much better work will be done if a plan is made for a unit and then the unit taught before another plan is attempted. Ideally this should be the procedure in each case.

If each student teacher is going to work individually on these plans, or do other individual work, much of the spare time and energy of the critic teacher will be consumed. The load of the average critic teacher is already far in excess of what it should be. One way to handle the difficulty would be to have the college teacher who teaches the methods or professionalized subject matter courses handle the conferences while the student teachers are working on the teaching plans. He should be an expert in this field, he should know what the student has done in the methods course, and he should keep in close touch with the observation and practice teaching courses and the pupils in the training school. However, the teaching plan also should be submitted to the critic teacher. The student should then teach the unit under the supervision of the critic teacher. The work of the critic teacher will be limited to directing the teaching and suggesting such changes that seem to be advisable because of the class conditions and of other considerations that could not be seen before.

During this entire teaching period special attention and thought should be given to desirable changes in the plan and to changes that might affect other plans to be made on different topics. This is necessary if the student is to get the most out of the course and be able to use the plan to the greatest advantage in coming years. Upon the completion of the unit the student should rewrite it for future use, considering the suggestions made during teaching. It would be desirable also if this work could be directed by the college teacher, as the actual outcomes could be recorded for comparative purposes when the unit is taught again. The student should then be able to make the plan for the next unit he is to teach much better and more easily than for the last one. This should again be done under

the direction of the college specialist and taught and revised in turn. During the making of the first plan the student should observe and take part gradually in the classroom activities in order that he may be ready to begin teaching when the class is ready for the unit he is planning.

If the entire participation and practice period is spent in this planning-teaching-revision procedure, it would seem reasonable to suppose that the student teacher would at least have a definite method of attack for the problems of actual teaching.

CHAPTER V

Samples of Teaching Plans of Units in Junior High School Mathematics

In Chapter II the form of the teaching plan was set up and explained. In the present chapter several examples of the teaching plan constructed from units in junior high school mathematics are presented. These plans illustrate the use of the teaching plan in several fields of mathematics included in the junior high school curriculum. This course in mathematics usually includes subject matter from informal geometry, social-economic arithmetic, algebra, trigonometry, and in a small number of cases, a unit in demonstrative geometry in the ninth grade.

Four of the six plans included are made on actual course of study units. Plan I, on *Insurance* is made for use in the state school system of Oklahoma and is built on a unit in the adopted textbook (10) for the eighth grade. The unit is taken exactly as it appeared in the textbook. Additional items were included only in the supplementary material and in the suggested activities.

Plan II, on *How to use Equations,* is made for a unit from the Course of Study for the Tulsa Public Schools, 1930, Tulsa, Oklahoma. This unit usually furnishes an introduction to the algebra that is given in the eighth grade of the junior high school.

Plan III, on *Practical Indirect Measurements* gives an introduction to trigonometry for the eighth grade. It is also made on a course of study unit used in the Tulsa schools.

Plan IV, on *Square root, Radicals, and Exponents,* is on a formal stereotyped unit. It is taught in grade 9-A and is in a large measure the same as the unit that has been taught for many years. It is also made on subject matter from the Tulsa Public Schools Course of Study.

Plan V, on *How to Measure and Draw Angles*, is made on material selected and organized by the writer. The unit furnishes an introduction to informal geometry and is usually taught in the seventh grade.

Plan VI, on *Demonstrative Geometry for the Ninth Grade,* is also made on subject matter selected by the writer. Neither this plan nor Plan V were tried in the classroom. They illustrate the use of the plan equally well, however.

Teaching units in junior high school mathematics might be placed in two classes, the formal or stereotyped unit as the one on Square root, Radicals, and Exponents and the newer type of teaching unit which is composed of subject matter new to these grades. This is illustrated by the units on Insurance, Equations, and Demonstrative Geometry.

In dealing with formal and stereotyped units, the teacher will probably know the subject matter well and have a clear idea of the whole before the unit is begun. He probably knows also the skills and what items of knowledge he will want the pupil to have when the unit is completed. It is possible to know these and many other things necessary to good teaching because of the formal organization of the subject matter of the unit, and the fact that the unit has been in the course of study of the schools for many years, substantially as it is now. For these reasons suggestions in the plan dealing with teaching procedure and the like would probably seem less original and less helpful to the teacher. Nevertheless, suggested teaching procedure should follow the line that experience and judgment indicate as most likely to give desired outcomes.

It is not so likely, however, that the teacher will know these essentials of good teaching when dealing with what has been called the new type of teaching unit. Many times the teacher knows too little about the subject matter because it is not in the secondary school or college curriculum. The method of organization and teaching presents other problems about which there are still questions. Selecting desirable objectives is another problem about which there is much uncertainty. A well constructed teaching plan, whether made by the classroom

teacher or by someone else, would at least give suggestions about all of these things and many others. It would undoubtedly be of value to most teachers if used properly. The value of the teaching plan of a unit would decrease appreciably with the experience of the teacher.

In view of the above discussion, it would seem reasonable to expect the teaching plan for the formal units of work to be of less value than it would be for those composed of the newer and less formally organized material.

Teaching Plan No. I

Section I. Identification

Teacher: ..... Grade: *8* Date: .....
School: ..... Subject: *Mathematics*
Title: *Meaning of Insurance* Duration: *Three weeks*

Section II. Subject Matter Outline

Insurance

A. The importance and nature of insurance
   1. Fundamental meaning
   2. Kinds
      a. Property
      b. Personal
B. Property insurance
   1. Fundamental meaning
   2. Kinds of property insurance
      a. Fire, tornado, lightning, automobile, etc.
      b. Fire insurance
         (1) Fundamental meaning
         (2) Words used in fire insurance
         (3) Tables of rates for businesses and dwellings
         (4) Computing fire insurance premiums using tables
C. Personal insurance
   1. Fundamental meaning
   2. Kinds
      a. Life, accident, health, etc.
      b. Life insurance
         (1) Fundamental meaning
         (2) Some words used in life insurance
         (3) Four common kinds of life insurance policies

Outline for Supplementary Problems

A. For those who will need more drill
   1. Problems on fire insurance
   2. Problems on life insurance

B. To care for individual differences
   1. More difficult problems on fire insurance
   2. More difficult problems on life insurance

Section III. Pupil Inventory

A. Inventory items

	Number of pupils				
	E	VG	G	F	P
1. General intelligence					
2. Previous knowledge of the unit					
3. General social background					

Note: E=excellent, VG=very good, G=good, F=fair, P=poor.

B. Occupation of parents

	No. pupils
1. Day laborers, domestic help, etc.	
2. Artisans, mechanics, carpenters, etc.	
3. Lawyers, doctors, farm owners, etc.	

Section IV. Aims or Objectives

A. General aims.
   1. To interest the children in the social values of arithmetic, and to give them knowledge, skills and attitudes regarding these needs.
   2. To develop the habit and power of quantitative thinking.

B. Specific aims (for fire insurance).
   1. Knowledge.
      a. Fundamental meaning of insurance
      b. Fundamental meaning of fire insurance
      c. Why companies can pay insurance
      d. The meaning and spelling of the following words and significant phrases.

insurance policy	good and poor fire
property insurance	protection
fire insurance	premium
insurer	annual premium
insured	business property
policy	dwelling house
rate of insurance	1-year premium
risk	

2. Skills

To develop the following abilities:

a. To read from an insurance table the rates of insurance for various types of businesses.
b. To compute fire insurance premiums for business houses and business house contents for 1-year periods.
c. To compute premiums on dwelling houses for one, two, three, four and five year periods.
d. To find what the insurance company would have to pay in case of fire.
e. To compute premiums when property is insured for a certain part of its value, as 80% etc.

3. Attitudes and appreciations.

a. To appreciate the fact that insurance is a protection for which we pay a reasonable sum, figured accurately by experts.
b. The responsibility of every one to be careful about fires in general because of
   (1) Danger to others' property as well as own.
   (2) Increased insurance rates for the community.
   (3) Loss to the insurance company.

C. Specific aims (for life insurance).

1. Items of knowledge

a. Fundamental meaning of life insurance.
b. Fundamental meaning of 4 types of life insurance.
c. Meaning and spelling of words.

personal insurance	ordinary life policy
policy	endowment policy
face of policy	limited payment policy
premium	term policy
insurer	20-payment policy
insured	20-year endowment policy
policy holder	5-year term policy
beneficiary	cash surrender value
dividend	loan value

2. Skills,

To develop the following abilities:

a. To read a table of rates for the premium on $1000 for any age and any kind of policy.
b. To tell from a table the cash surrender value of a policy of any size at the end of a given number of years.
c. To find the premium, having given the age, amount of the policy and type of policy.

d. To compare the amount paid in and the amount received when a 20-year endowment policy runs its full term.
e. To figure the net cost of insurance that carries dividends.
f. To find the amount of insurance it is possible to carry with a certain amount of money available, as $500 per year.

3. Attitudes and appreciations
a. To appreciate the value of insurance as an investment.
b. To appreciate the responsibility of carrying insurance for those who are dependent on one for support.
c. An appreciation of a young man's responsibility of carrying insurance to cover debts, even though he may have no dependents.

Section V. Suggested Activities and Exercises

A. Activities (for fire insurance)

1. Classify their own homes according to the table on page 357*, and estimate its value and figure the fire premium.
2. Classify their father's business house or his barn or stable and figure the fire insurance premium.
3. Form a mutual insurance company against the loss of fountain pens, in which each one is assessed a certain percentage of the value of pens lost by any member of the company. Imagine various pens have been lost and let the class decide the amount of insurance the owner can collect under his contract and the assessment each member must pay. Bring out the fact that a large number of members would reduce the assessment on any particular member.
4. Secure old or sample fire insurance policies and discuss the terms found and let the pupils fill in the policies for their own homes.
5. Insurance for solved problems. In this suggested way to "play insurance" Thorndike (52:119-120) outlines an activity that should stimulate the interest of the child as well as give him the meaning of most of the fundamental terms of insurance. A test of 20 problems is given, with 5 extra problems included. Each

* Page numbers throughout this unit plan are from the textbook in use, Fowlkes and Goff, Modern Life Arithmetic, Advanced Arithmetic. New York: The Macmillan Company. 1929.

pupil is given the option of solving one of the extra problems to "pay" for insuring his test paper against "Fire". If he turns in the extra problem, he is given a contract or a policy (these can be made out by the pupils) by one of the pupils called "Insurance Company" stating that his 20 problems are insured against "Fire" for a term of five hours. When every one has turned in his paper, one pupil called "Fire" comes up and with his eyes shut picks out a test paper and destroys it. If the owner has an insurance contract or policy, "Insurance Company" must pay him 20 problems from the ones paid him as premiums. The owner of the destroyed paper may then give these to the teacher and does not have to do the 20 problems over again. If he is not insured, he must remain after school and rework the 20 problems. Solving the extra problem would give the pupils protection against the danger of the destruction of his paper by "Fire".

B. Activities (for life insurance).

1. Form a mutual insurance company in which each agrees to pay a specific amount on the death of any member. Bring out the fact that there would have to be a large number of members in order that a small fee from each would be sufficient for the family of the one who had died; also bring out the disadvantages to the younger members of the group and point out who would do the clerical work, and the like. (2:39) Compare with a regular insurance company.
2. Find rates from local agents and figure the premiums different people would have to pay.

C. Exercises

1. For use in regular class work
   a. For fire insurance
      1. Questions on page 350 dealing with rate table.
      2. Questions on pages 352-3 dealing with methods of solving problems.
      3. Problems on pages 353-4, numbers 1-8
   b. For life insurance.
      1. Questions on page 360 regarding the rate table on page 359.
      2. Questions on 361 regarding the table on page 360.
      3. Problems 1-8 pages 361-2 on finding life insurance premiums.
      4. Problems 1-26 pages 363-365, on review.

2. For use as supplementary exercises to care for individual differences.
   a. For remedial purposes

(These problems are of the same nature as those of the regular unit)

PROBLEMS ON FIRE INSURANCE

1. Using the tables in the book (pages 350-351) fill in the blanks.

Dwelling or business	Kind	Value Bldg	Value content	Premium
Boarding house	brick, shingle	$5000	$3000	
Bank		5000	25000	
Barn	frame, shingle	2000	25000	
Hotel		15000	8000	
Motion picture		4000	1000	

2. Using the tables in the book (pages 350-351), fill in the blanks.

Dwelling	Kind	Value Bldg.	Value content	No. yrs.	Premium
Apartment house	Brick, metal	$8500	$1000	3	
Club house	brick, shingle	5000	2000	5	
Stable	brick, metal	4500	2000	2	
Double dwelling	brick, shingle	7500	3000	4	
Boarding house	brick, shingle	2000	5000	1	

3. Mr. Jones has a house insured for $1800 at the rate of 30c a $100. What is the amount of his premium each year?
4. What is the cost of insuring a building valued at $32,000 if it is insured for 80% of its value at the rate of $0.15 a $100?
5. How can a company for a small premium of $12 afford to pay the owner $2400 in case his barn is burned?

PROBLEMS ON LIFE INSURANCE

1. Using the table in the book (page 359), fill in the blanks.

Name	Age	Kind of policy	Amount	Premium
J. R. Smith	20	ordinary life	$2000	
M. C. Newton	40	5-year term	5000	
J. R. Hansen	45	20-payment life	1000	
Carl Abbott	25	ordinary life	10000	
M. B. Milo	50	20-year endowment	15000	
R. A. Bell	35	20-payment life	4000	

2. Ralph Foss was insured at the age of 30 for $3500 on the ordinary life plan. What was his annual premium? He paid premiums for 12 years. What was the total amount

of his premiums? He died shortly after paying his twelfth premium. How much did his beneficiaries receive?

3. Fred Furst was insured at 30 for $3500 on the 20-year endowment plan. Answer questions like those in question 2.
4. John Keel insured at the age of 20 by the ordinary life plan. He died at 65. What is the difference between the amount of premiums paid and the amount of insurance received? (face of the policy $5000) Omit interest.

For the Better Group of Pupils

(The outline of this work is included in Section II above)

More problems may be added if needed.

More difficult problems on fire insurance

1. Read supplying the missing numbers:
   a. Mr. Andrew's house, worth $4000, is insured for ¾ its value, or for $.....
   b. Mr. Filby's house, worth $4000, is insured for 80% of its value, or for $.....
   c. Mr. Crow's house, worth $6000, is insured for $4500, or for .....% of its value.
2. What expense would an insurance company have in securing business?
3. Careful people keep an inventory of their property. How would this be used in case of fire?
4. How much insurance can you buy for $30 if the rate is ½%?
5. A factory worth $60000 is insured for 5/6 of its value at a rate of $2 a $100. If a fire had caused a total loss after three premiums had been paid, how much would the owner have lost?
6. A man insures his barn and contents, valued at $2800, for ⅝ its value. At 60c a $100 of insurance, what premium does he pay?
7. A man insured his home for $3500 for a period of three years at 60c a $100. Two years later a fire caused a damage of $3000. How much did it cost the man for each $100 of settlement.

More difficult problems on life insurance.

1. Mr. Hunt intended to take out a 20-payment life policy when he was 20. He neglected to take out insurance until he was 30. How much more must he pay during the 20 years than if he had insured at the age of 20?
2. A life insurance company sold a man a $2000 policy for $75 a year. After making the sixth payment the man died.

How much more was received by the beneficiary than would have been received if the premiums had been deposited in a savings bank at 4% interest, compounded semi-annually?

3. How does a 20-year endowment policy differ from a 20-payment life policy? Why are endowment rates higher?
4. Mr. Alden carried a 20-payment life policy for $1000 for 10 years. His yearly premiums were $36.22. During the ten years the dividends amounted to $61.69. Find his average yearly payment.
5. At the end of 10 years Mr. Alden (problem 4) dropped his policy. Its cash value was then $255.78. How much a year on the average had his insurance cost him.
6. A man 27 years old decides to put an additional $100 a year on insurance. Find the nearest number of thousands of insurance he can carry for that amount in each of the four types of policies.

Section VI. Teacher Procedure and Means

Outline

A. The use of the teaching plan during teaching.
B. Detailed procedure in teaching the unit.
C. The use of the supplementary exercises and problems.

A. The use of the teaching plan during teaching.

If the teaching unit plan is to serve the purpose for which it is intended, i. e., the improvement of instruction, the teacher must be thoroughly familiar with it before beginning the unit, and use it throughout the teaching. In order to make it easier for the teacher to do this, we are trying to set out in some detail a suggested procedure in the use of the teaching plan. Becoming thoroughly familiar with the teaching unit and the teaching plan of the unit is the responsibility of the teacher. It is felt, however, that the following suggestions about the use of the various parts will probably be helpful.

1. *Pupil inventory.* To be able to teach well the teacher must know his pupils. The material for this section should be very carefully collected for the pupil to be taught. If any of the data is not available, make estimates as closely as possible and check with opinions formed by other teachers. Much of the teaching should be guided by the facts tabulated in this section. If the pupils are found to be extra good intellectually, more and better work will be expected and different methods

of teaching used. If the pre-test shows that they know many items included in the unit, these will naturally be stressed less. The pupil inventory will affect the teaching in many other ways also.

This should tend to isolate certain groups of the class who will need special attention. Not the least important of these will be the best members of the class. Definite plans are made to take care of this group. Material for their special benefit has been included and will be discussed later in the plan. The teacher will have to keep the group definitely in mind, however. There will be other groups so isolated for whom special planning will be necessary.

2. *Subject matter outline.* The purpose of the subject matter outline is to give the teacher a view of the unit as a whole. It enables him to see the entire structure and to think of it in a unified way. The teacher will be unable to make the pupils see the unit as a whole if he does not see the unity of it himself. This would be possible, of course, without actually having the outline before one, but it is not so likely.

The outline goes further, however, in that it summarizes the supplementary material for those who need more practice and for those who have more time than needed for them to acquire the suggested knowledge and skills. Definite effort should be made to retain unity with the preceding material. This supplementary material will be found in Section V.

3. *The aims.* Any good teacher will generally see the worth of any valid set of aims, but this does not mean that these aims will be attained unless one sets out definitely to attain them. Unless one has carefully thought them out, listed them, and kept them constantly before him while teaching, some of them are sure to be lost. This is especially true of a teacher who is not so well trained, or who has had little experience. Before beginning any unit, then, the teacher should carefully study the aims, see if he agrees with them, and see if there are others which should be added. The teacher should never hesitate to include other aims if he has such a conviction.

In the case of attitudes and appreciations the plan may legitimately go beyond the textbook. While attitudes and appreciations may be included in the textbook, that which the pupil gets will depend to a large extent on the attitudes and appreciations of the teacher, and to some extent on local conditions. A teacher, then, will honestly try to instil only those attitudes and appreciations which he feels are just and worthwhile. More will be accomplished toward this end by the example and enthusiasm of the teacher than by textbook examples.

4. *Activities and exercises.* "Exercises refer to those particular portions of the subject matter to which the laws of learning and drill are particularly applicable; activities refer to the whole hearted, purposeful activity, or at least a large experience that will involve subject matter from several different fields or several parts of one field. Consequently, exercises involving only the manipulation of facts and ideas too often lack reality for the pupil. The activity is designed to bring reality back to the classroom." (3:17) The activity intends to make the work clearer to the pupil, though it will naturally vary with local conditions. A number of activities are suggested in the plan and may be used to advantage in introducing the unit and in keeping the interest of the class. The teachers should choose the ones that will fit the local conditions best. It is not necessary to choose from the list suggested in the plan. Special suggestion as to their use will be made later in this section. The use of the activity may be considered as primarily the responsibility of the teacher in introducing and carrying on the unit.

Little need be said of the exercises; they will consist of questions and problems in the book and a group of supplementary problems for extra drill. The teacher should feel free to add others if he feels that the pupils are not getting the work. The use of exercises as remedial and supplementary material will be more fully discussed under C below.

5. *Correlations with other subjects.* The correlations with other subjects and within the subject will add to the interest of the unit as well as add much to the pupils' knowledge. Any

correlation that is attempted should be due to a real connection and not forced, merely for the sake of having a correlation. The ones given do not complete the list by any means; the teacher should be constantly on the lookout for their possibility.

6. *Bibliography.* The bibliography is made rather long, and it is not expected that many pupils or teachers will actually refer to all of the books listed. Many of the books will not be available in some schools; hence it seems advisable to lengthen the list. There will be an occasional pupil or teacher who may want to go deeply into the unit, and he should be well provided for. It is also not expected that any teacher will limit himself to the bibliography given. If other books are available, the pupils should have the advantage of their use.

7. *The Testing Program* bears a close relation to the aims and also to the subject matter outline. Tests should not only be used to tell what the pupils have learned, at the end of the period, but should also be used as teaching devices. The exact place in which to use the tests will be indicated further along in this section and also the use to be made of the test scores. The teacher should be familiar with the type of questions and what the tests ask for. The tests are made to test most of the measurable aims of the unit.

B. Detailed procedure in teaching the unit.

1. *Introduction to the unit giving the fundamental meaning of insurance.* In introducing a new topic like this it is the duty of the teacher to make the fundamental meaning as clear to the pupil as possible, using any means within his power. If some class activity can be set up which will make the meaning clearer to the pupil, it should be done by all means, even though such activity is not listed in the textbook. In the present unit an activity or class discussion can be set up that will probably greatly aid the pupil in learning the fundamental meaning of insurance. Let the class imagine that it forms a mutual insurance company of some sort. In some schools companies are actually formed to insure the members against the loss of fountain pens or other such property which most of the students have. The members agree to pay 5c or 10c to any

other member of the group whose pen is lost. By a discussion of this they can easily see that if the company has a large number of members the fee would be less, and that if there are not too many losses it would be an advantage to belong to such a group. They will also see the need and the expense of having some one to handle the collection etc., and also someone to decide when a pen is really lost in order to prevent some unscrupulous member defrauding the company. This will be analogous to the adjuster of a fire insurance company. If the teacher can make this real to the pupils and keep their interest in it, they will get a much clearer idea of the fundamental meaning of insurance than if they read a group of statements and definitions in the textbooks. A list of suggested activities will be found in Section IV, and the teacher is urged to use what he feels, under local conditions, will give him the best results. It is very important that the pupil be given this fundamental meaning, if possible, for it will add to the reality of the whole unit.

2. *Fire insurance.* a. *The fundamental meaning.* If the pupil has the fundamental meaning of insurance in general, it will not be difficult for him to see what is meant by fire insurance. It is merely reimbursement for a loss by fire rather than for loss by some other means. If someone's home has burned recently or if a house near the school has burned, it can probably be discussed to advantage.

b. *Teaching the meaning of the terms.* An excellent activity to use here, which will also help in A above, would be Activity No. 3 listed in Section IV, that of having the pupils inspect some old fire insurance policies or some sample policies. These can usually be had from the insurance agent in town, or sometimes pupils are able to bring them from their homes. The children might be asked to fill in the blanks using the values of their homes. This will involve a discussion of the meaning of many of the terms; more explanation will have to be given for some of the terms, as risk, good fire protection, and the like. In any case the children must know the meaning and spelling of these terms, if they are to be used to the greatest advantage in life and in the future work of the unit.

The pupils should understand that they will be tested on the meaning and spelling of all the words and phrases. When they have studied the assignment Test I, Section IX should be given. This is a new type test and may be given in a very short time and marked objectively. Meanings not understood should be studied by the pupils individually. If the test reveals that a majority of the class does not know the meaning of most of the words, the teaching should be done over.

c. *The tables of rates and how to read them.* In order to be able to solve the problems on insurance the pupils will have to understand the tables on pages 350 and 351 in the text. It will probably not be difficult if the pupil is first taught the meaning of the tables and why the rates vary. Explain this carefully, and then drill the pupils on finding rates for different kinds of buildings and contents and the like. No test is provided for this, since it is possible for the teacher to find by an oral drill much more quickly whether the pupils are able to find rates.

It is important for the teacher to remember that the concept of dependence is considered one of the most important concepts in mathematics. (53:251) It is more generally called the function concept. If this concept is to be thoroughly acquired, it will be necessary for the teacher to stress it many times in the school life of the pupils. The work with insurance tables is one place in which the function idea may be introduced to advantage.

d. *Teaching the use of these tables of rates in solving* problems. If the preceding work has been well taught, it will probably be rather easy for the pupils to solve the problems. To a certain extent the pupils' ability to solve the problems is a measure of the teacher's efficiency in teaching the first part of the unit. The solving of the problems involve the skills listed in Section I under Specific Aims. For the pupils who are unable to succeed with the problems it would be well to check their ability on each of these skills. This can probably be done most economically with the individuals during the supervised study periods.

When one feels that the pupils have learned this part, Test II should be given. This is a test on the part of the unit dealing with Fire Insurance. This test will cover the terms as well as the problems. The test is also a teaching device; and if the pupils miss many of the questions, they should be taught these things again. Additional problems and exercises for this purpose are given under Section V, as well as more advanced ones for those who have succeeded well in the tests and may have to wait a few days while the others catch up. This is more fully discussed below.

3. *Life Insurance.* The discussion of the teaching of separate parts of life insurance would be so much like the discussion under Fire Insurance that it will not be repeated here. If the teacher has studied and taught carefully through the above section on Fire Insurance, he will probably be able to apply the same technique to Life Insurance. Follow the same procedure in teaching the terms and the reading of the tables. In this case give Test III after the table on cash surrender or loan values has been taught in order that the pupil may have the meaning of all the terms. Follow the same procedure about reteaching those items which the tests reveal the pupils do not know. Test IV will be the final test on the unit and should be given at the end of the review exercise on page 363-5.

C. The Use of the Supplementary Drill Exercises and Problems.

1. *For remedial purposes.* The use of these problems and exercises has been mentioned above, but their efficient use demands careful consideration. If teaching is to be at all efficient, something definite must be done about the errors that the pupil makes. Too often the same drill is given again, which is uninteresting to the pupil, or, one may try to think, while in class, of suitable exercises and problems, and find it a hard task. Usually in either case he is not concentrating on the particular thing that the child needs. To do this efficiently the exercises and problems must be thought out beforehand. In the set of problems given in Section V, the difficulties of the teacher and pupils have been anticipated and problems are given to remedy the separate skills. The teacher will find it

definitely worth while to use these exercises and problems systematically.

2. *To take care of individual difficulties.* While the task considered in the above paragraph is usually called for because of individual differences of the pupils, it may be considered in general as material for a remedial program for any pupil who fails, for whatever reason. The program, if wisely used, will take care of any individual on the lower end of the scale. The neglected pupil in our schools, as one has so often heard, is the brighter one. While the slow one is catching up he is marking time, learning bad habits, and the like. It is much more essential that the group of problems and exercises for these pupils be thought out carefully if they are to serve best. Anyone who has tried giving a pupil from this group a problem offhand has surely found that in many cases he works the problems long before the teacher can settle one difficulty for a slower pupil. This means a potential disciplinary problem, unless one stops in the middle of an explanation and gives him some more problems. Now it seems that this type of pupil would profit more and be more interested if the teacher had a group of problems and exercises already prepared and thought out beforehand that would be hard enough and advanced enough to be worthy of his ability. These problems should represent an extension of the topics being studied, and if they could be given to the pupils in books with some explanation, it would be better. There is no way of telling how many will be needed; the teacher will have to take the responsibility of providing some of them, if the list given is not sufficient.

Section VII. Correlations

A. With other subjects
   1. *With science.* Many places will be found in this unit where references to science will be advantageous. The following instances are cited as examples:
      a. The inflammability of certain liquids and gases used about houses and stores, such as gasoline, benzine, various cleaning fluids, natural gas, and the like.
      b. Why certain types of wood burn more readily than others.

c. Why certain types of shingles will resist fire.
d. Something about spontaneous combustion, especially with reference to green feed in barns.
e. Why paint protects wood against fire.

2. *With the social sciences.* The following instances are cited as examples:
   a. Social value of the protection furnished by insurance.
   b. Value on insurance as an individual investment.
   c. Insurance may be considered as correlated with social science at all points, since it is a social science topic quantitatively treated.

B. Within the subject
1. *Interest.* Some connection between the subject of interest and insurance can be pointed out to advantage. For example in figuring the actual cost on insurance the interest on the money must be considered. This will involve both simple and compound interest.
2. *Business practice* includes some cases in which insurance plays a part. In buying automobiles on the installment plan, the buyer is obliged to have the machine insured, in most cases, to protect the seller. The same is true in buying a house through the building and loan companies.
3. *The study of thrift* is closely connected with insurance, since the buying of insurance is one of the best ways to save money and to have protection for one's family at the same time.

C. Within the unit
1. *The relation between fire insurance and life insurance* has been discussed above.

Section VIII BIBLIOGRAPHY

A. For teachers

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A short discussion of the method of introducing the topic of insurance to a class of children.

Bonser, Pickell and Smith. Practical Mathematics for Junior High Schools. Second Year. Chicago: Mentzer-Bush Company, 1924. pp. 57-64.

A good chapter on the topic of insurance.

Roantree and Taylor. An Arithmetic for Teachers. New York: The Macmillan Company, 1926. pp. 490-502.

A discussion of insurance which goes somewhat beyond the limit of the unit and is a good reference for the teacher who wants a fuller background.

Sone, J. C. The New Mathematics, Book III. New York: Benj. H. Sanborn Company, 1927. Chapter X.

A good chapter written for a ninth grade class which has had a good unit on insurance in the eighth grade. It has excellent material in it suitable for the better pupils and to give the teacher a broader background.

Taylor, E. H. Arithmetic for Teacher Training Classes. New York: Henry Holt Company, 1926. Chapter XVIII.

A short discussion of life insurance containing a short American Experience Table of Mortality. It is excellent for those who are not familiar with this type of table and its uses.

B. For Pupils

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Knight, Studebaker and Ruch. Standard Service Arithmetics, Grade Eight. New York: Scott, Foresman and Company, 1928. pp. 149-184.

Schorling and Clark. Modern Mathematics, Eighth School Year. Yonkers, New York: World Book Company, 1929. pp. 234-284.

Stone, J. C. Op. cit. Chapter X.

Strayer-Upton. Junior Mathematics, Book Two. Cincinnati: American Book Company, 1929. Chapter X, pp. 179-190.

Section IX. Testing Program

Test I.

(In each blank place the word that makes the best sense in the sentence.)

1. A written agreement in which the company agrees to pay a person a certain amount in case of loss or damage is called an _____ _____.
2. A protection against loss or damage to property is called _____ insurance.
3. If this loss is by fire, this protection is called _____ _____ insurance.
4. The insurance company is called the _____.
5. The person protected is called the _____.
6. The money paid for insurance is called the _____.

7. The amount paid for $100 of fire insurance is called the_____of fire insurance.
8. A brick house built near a fire plug, a reasonable distance from other buildings, stocked with goods that do not catch fire easily, and not too far from a fire station may be expected to have a_____insurance rate.
9. Wooden buildings with shingle roofs stocked with goods that catch fire easily, close to other buildings, not near a fire plug or fire station, and the like may be expected to have_____insurance rates.

Test II.

(In each of the blanks of the first three sentences place the word that makes the best sense. In the other problems solve on scratch paper and place the answers in the blanks.)

1. The amount paid for $100 of fire insurance is called the_____ of insurance.
2. Protection against loss or damage to property is called _____insurance.
3. The person protected is called the_____.
4. Fill in the blanks below using the tables in the book (pp. 350 and 351).

Property	Value		Kind	Annual	2-yr.
	Bldg.	Content		prem.	prem.
Groc. store	$2000	$5000			
Motion piet.	5000	10000			
Barn	1000	3000	frame, metal		
Garage	2000	5000			
Pvt. dwelling	5000	2000	brick, shingle		

5. Mr. Scott insured his private residence, which is a brick house with a metal roof, for one year for $12000 and he also insured the contents (household goods) for $8,000. The total premium he had to pay was $_____.
6. Mr. Halverson insured his grocery store building, worth $30,000, for 80% of its value for a period of 5 years. His premium was $_____.
7. If the building is destroyed within the five year period, the insurance company must pay $_____. Mr. Halverson's loss counting the cost of insurance was $_____.

Test III.

(In each blank place the word that makes the best sense in the sentence.)

1. The protection against loss of income caused by illness, accident, or death is called_____insurance.

2. The arrangement by which an insurance company agrees to pay a certain sum of money to a person's family upon his death is called _____ insurance.
3. The written agreement to do this is called the _____ _____.
4. The amount of money paid to the family of the insured is the _____ of the policy.
5. The amount of money paid for life insurance is called the _____.
6. The life insurance company is the _____.
7. The person whose life is insured is the _____.
8. The person who receives the money from the insurance company is called the _____.
9. Money paid back to the insured during the time the policy is in force is called a _____.
10. When a man pays a premium each year until his death, no matter how long, it is called an _____ life policy.
11. When a man pays his premium for a certain number of years, at the end of which time the company has agreed to pay him the face of the policy, it is called _____ _____ policy.
12. When a man has to pay only twenty payments and after that the policy is in force until his death, it is called _____ _____ policy.
13. When a man agrees to pay premiums for a period of only 5 or 10 years and after that the policy is not in force, it is called _____ _____ policy.
14. The amount that an insurance company will pay the policy holder who stops paying premiums is called the _____ or _____ value.

Test IV. (Unit Achievement Test)

(Solve the problems and place the answer called for in the proper blanks.)

1. Fill in the blanks using the tables in the book, (pages 350-351).

Property		Value		1-yr.	2-yr.	3-yr.
	Kind	Bldg.	Cont.	prem.	prem.	prem.
Barn	frame, metal	$3000	$1500			
Hotel		10000	5000			
Garage		4000	6000			

2. Fill in the blanks, using the tables in the book, (page 359).

Name	Age	Kind of policy	Amount	Premium
J. L. Norton	25	5-year term	$5000	
O. M. Roe	35	ordinary life	4000	
R. A. Bell	40	20-year endowment	2000	

3. Mr. Ray's dwelling house is valued at $5000 and he has it insured for 80% of its value at 22c a $100. The yearly premium is $_____.
4. Mr. Smith took a 20-payment policy at the age of 25 for $4000. If he lives to pay it out, he will pay the company $_____. If he dies at the age of 55 would it have been better for him to have taken an ordinary life policy? Yes_____ No._____.
5. Mr. Horner takes an ordinary life policy for $5000 which costs him $28 a $1000 but pays an average yearly dividend of $8.14 a $1000. The average yearly cost of the policy was $_____.
6. A man 25 years old finds he can save $125 a year and decides to buy insurance with it. He will be able to buy to the nearest $1000 $_____ of ordinary life policy; $_____ of 20-payment life policy; or $_____ of 20-year endowment policy.

TEACHING PLAN NO. II

Section I IDENTIFICATION

Teacher: _____ Grade: 8 Date:_____
School: _____ Subject: *Mathematics*
Title: *How to use Equations* Duration:_____

Section II. SUBJECT MATTER OUTLINE

How to Use Equations

A. Introduction to the meaning of the equation
B. The nature of equations
   1. Development of the fundamental laws of equations or the axioms
C. Solving equations by subtraction
D. Solving equations by division
E. Solving equations by addition
F. Soving equations by multiplication
G. Checking in solving equations
H. Solving equations requiring the use of more than one law

Outline for Supplementary Work

A. For remedial purposes.
   1. Simple equations
      a. requiring use of the subtraction axiom
      b. requiring use of the division axiom

c. requiring use of the addition axiom
d. requiring use of the multiplication axiom

2. Equations requiring the use of more than one rule to solve
3. Written problems solved by equations

B. For the better group of pupils

1. More difficult problems of the type 2 above
2. Written problems using equations covering a wider range
3. The transformation of formulas
4. Formulas and equations of percentage and amount

Section III. Pupil Inventory

(See Section III Teaching Plan No. I.)

Section IV. Aims or Objectives

A. General Aims

1. To help give the pupils some realization of the powers and range of mathematics in the solving of many sorts of problems.
2. To show the function idea as related to equations.
3. To show the value of the equation as an aid in using formulas.
4. To help the pupil in his "growth of understanding" and ability to understand.

B. Specific Aims

1. Items of knowledge
   a. words and significant phrases
      equation
      members or sides of the equation
      laws of equations
   b. principles
      (1) if the same number is subtracted from both sides of an equation, the sides are still equal
      (2) if both sides of an equation are divided by the same number, the sides are still equal
      (3) if the same number is added to both sides of an equation, the sides are still equal
      (4) if both sides of an equation are divided by the same number, the sides are still equal
      (5) that more than one of the principles must sometimes be applied in order to solve an equation
      (6) that a solution may be checked by substitution of the value of the unknown in the equation

2. Skills
   To develop the following abilities:
   a. to solve equations using each of the four principles or axioms
   b. to solve equations in which two or more of the axioms will have to be used
   c. to check equations by substituting the value of the unknown
   d. to translate verbal problems into equations
   e. to use the three abilities a, b, and c in solving practical problems
   f. to interpret the results and get the answer of a problem
3. Attitudes and appreciations
   a. to appreciate the value of the equation as a tool for solving problems
   b. to appreciate the fact that the equation coupled with the formula form the part of algebra which is most often used by people at large
   c. the attitude or interest in learning something about one of the things that the scientfiic world uses most

Section V. Suggested Activities and Exercises

A. Activities.
   1. Try to get the pupils interested in finding formulas and equations in books and magazines they read. Articles dealing with mechanics and radio very often have formulas in them.

B. Exercises.
   1. For use as regular class work
      (The exercises to be used in teaching will be found in the textbook. (38)
   2. For use as supplementary exercises to care for individual differences
      a. for remedial purposes

(The problems listed below are of about the same difficulty as those appearing in the unit.)

Equations

1. Requiring the use of the subtraction axiom

$n+3=9$	$7+n=12$	$n+7=14$
$n+6=10$	$8+n=14$	$n+8=22$

2. Requiring the use of the division axiom

$3x=15$	$3n+7=25$	$5x+7=42$
$7x=42$	$5x+2=32$	$6x+8=50$

3. Requiring the use of the addition axiom

n—3=5	n—2=6	x—¼=2	8—n=5
n—2=6	n—½=2	n—⅓=3n	n—7½=10

4. Requiring the use of the multiplication axiom

x/2=8	x/6=9	x/7=3.4	x/7=5/6
x/3=9	n/2=3.5	x/9=1.2	x/8=2/3

5. Requiring the use of more than one axiom

3x+3.5=8	2x/7=2/3	3x/5=1½
2x—5=12	5x—3=42	2x—4½=3

Problems solved by the use of equations

1. John wishes to cut a rope 18 feet long into two pieces such that one piece will be twice as long as the other. Find the length of each piece.
2. Frank has a piece of wire ten inches long which he wishes to cut into two pieces so that one piece is two inches longer than the other. Find the length of each piece.
3. In a class of thirty-two pupils, there are four more girls than boys. How many boys are there?
4. At an entertainment there are twice as many children as adults. There are 81 in all. How many adults are there?
5. Mary said, "I am thinking of a number. If I double it and add 3, the sum is 19." Of what number was she thinking?
6. Henry said, "My garden is 10 feet longer than it is wide. It takes 100 feet to enclose it." Find the width.

b. For the better group of pupils

More difficult equations

2½x+7=32	7x+3x+7=24	8x—5x—6=8
12=2½x+2	6x—6=126—5x	7x+2x—15=48

More difficult problems solved by equations

1. Each time John makes a deposit on his savings account his father adds three times the amount. John now has $48 in the bank. How much did he save alone?
2. The circumference of a circle is 2 *pi* times the radius, and *pi* is 3.14. Find the radius of a circular plot whose circumference is 125½ feet.
3. In a business enterprise Mr. Wright agreed to furnish 1.5 times as much money as Mr. Carter. If $8,300 is required, how much money must each man furnish?
4. If fertilizer contains 4% of nitrogen, 10% of phosphorus acid and 8% of potash, how many pounds of each are there in 15 pounds of fertilizer?

5. A merchant paid $28 for a suit. At what must he mark it to give a margin of 30% of the cost?
6. During a thrift stamp campaign, Frank sold twice as many stamps as James did, and Ralph sold as many as both together. Together they sold $240 worth. How many were sold by each?

Solving formulas

In $A=bh$, find h when $A=80$, and $b=20$.
In $A=\frac{1}{2}bh$, find b when $A=80$, and $h=16$.
In $C=2\pi r$, find r when $C=314$. (use $pi=3.14$)
In $d=2r$, find r when $d=600$.
In $A=\frac{1}{2}h(B+b)$, find h when $A=600$, $b=40$, and $B=60$.
In $V=lwh$, find w when $V=900$, $l=75$, and $h=6$.

Change the subject of the following formulas as required:

$A=bh$, solve for b.
$A=\frac{1}{2}bh$, solve for h.
$C=\pi d$, solve for d.
$V=\frac{1}{3}\pi r^2h$, solve for h.
$C=2\pi r$, solve for r.
$i=prt$, solve for r.
$V=lwh$, solve for h.
$V=\frac{1}{3}bh$, solve for b.

Formulas and equations of percentage and amount

1. At a 15% discount sale Helen bought goods for a dress for $9.20. What was the regular price of the material?
2. Miss Vail receives 6% discount on a bill. If the net amount of the bill is $42.30, find the original amount of the bill.
3. Mabel had some money in the savings bank on which she received 4% interest. If she received $12 in interest last year, what amount did she have in the bank?

Solve and check

4. $x-\frac{1}{2}x=120$.
5. $x-\frac{50}{100}x=\$390$.
6. $x-\frac{75}{100}x\ \$720$.
7. $p-\frac{24}{100}p=\$380$.
8. $p-\frac{81}{100}p=\$410$.
9. $x-\frac{1}{5}x=\$11.12$.

Section VI. Teacher Procedure and Means

Outline

A. The use of the teaching plan during teaching
B. Detailed procedure in teaching the unit

C. The use of supplementary drill exercises and problems

A. The use of the teaching plan during teaching.
(For a discussion of A see Teaching Plan No. I)

B. Detailed procedure in teaching the unit.

1. *Introducing the unit.* After obtaining from the pretest, described above, some idea of the pupils' knowledge of equations, the teacher's aim in the introduction of the unit should be, to give the pupil a brief outline or preview of the unit, to show something about the use of equations in actual situations, and most of all to make the pupil want to study about them.

To show the pupil the value of the equation, such an introduction as the following may be used:

"*The value of formulas and equations.* A large reservoir built to furnish electric power to Kansas City and St. Louis, was recently completed by placing a dam across the Osage River at Damsite, Missouri. Before a yard of concrete was poured or a spadeful of earth moved the engineer who planned this dam had to make many measurements and calculations. Many of these measurements had to be made indirectly, as it was impossible to measure the height of the cliffs on either side, and the like, by the use of an ordinary tape measure. The engineer made some measurements on the ground and then measured an angle to the top of the cliff with a surveyor's transit and was able, by the use of a formula he knew, to compute the height of the cliff very accurately. This formula is sometimes written,

$$h = a \times \text{tangent } A.$$

"You are not able to understand this formula now, but it will be explained later in this book and you will be able to make calculations yourself with simple instruments which may be made in class or at home. There are many problems in school and in life that may be solved quickly by a formula, if you know the one to use. If you are able to use a formula or a rule, you do not have to spend time to figure or reason out the problem.

"Many of the calculations which the engineer had to make while he was drawing the plans for the dam were made by using simple rules or formulas which you know. For example, the foundation consisted of concrete laid beneath the ground forming a rectangular block. The volume of such a block could be found by *multiplying the length by the width by the height.* This is a rule which you have all learned and many of you will remember that the formula is

$$v = l \times w \times h.$$

It is usually written without the multiplication signs, that is

$$v = lwh.$$

We could say that the formula is but a short way of writing the rule. Sometimes we say that the formula is a 'shorthand' way of writing the rule."

The actual introduction of the work on equations should be as a kind of shorthand method of solving problems and a sort of game used to find an unknown number. The use of letters for these unknown numbers comes naturally, and the pupil is able to solve many simple equations. Some work with formulas is assumed in the discussion of methods of teaching equations.

2. *Solving equations intuitively.* After equations have been introduced by the puzzle or game problems mentioned above, the pupils should be given some practice in solving simple equations intuitively. That is, give them the equations and ask what the unknown number is. They can easily solve the simple ones, as $x+6=10$, $n-4=8$, $3n=12$, and the like. This is done without having in mind any of the equation laws or axioms. They can just "see" what the answer is. The pupils will be interested in this type of work and should solve a number of such problems.

3. *The nature of the equation.* Equations which are not susceptible to such an easy method of solving should be given next. Equations, such as $4\frac{1}{3}x=13$ and the like, will show the need of more definite methods of solving problems. The pupil has naturally assumed certain things about equations in making simple solutions. For example, when given $x+6=10$, he has

subtracted 6 from 10 which is equivalent to assuming that x is 6 less than 10. (See Everett, Reference 1, Bibliography, Section VIII). He should be led to make these assumptions formally and to understand their meaning. This amounts to seeing and stating that both sides of an equation are equal, that the value or size of x, the unknown, depends on the size and relations the known numbers bear to it, and to the fact that the same number may be added to, subtracted from, divided into, or multiplied by each side of the equation without affecting the equality of the two sides.

a. *Solving equations by substitution.* In this unit the nature of the equation is developed by the use of the ordinary see-saw or teeter board. The development will not be repeated here, but an attempt is first made to show why the two sides must be equal, and that if the same number or same weight is taken from both sides the see-saw would still balance. This leads to the rule or axiom, that if the same number is subtracted from both sides of an equation, the sides are still equal. Exercises given should contain some with large numbers and decimals so that the pupils may see that a rule is needed to solve some equations.

b. *Solving equations by division* is easily understood by the pupil. A balance or see-saw may help them to understand. Children solve the equations easily but often mechanically. This may be seen if an equation like $8x=2$ is given; the pupils divide 8 by 2 and get 4 for an answer instead of $\frac{1}{4}$. Watch carefully for this and be sure that the pupil understands the process.

c. *Solving equations by addition.* Pupils can often solve an equation like $x-2=10$, but fail to see that adding 2 to both sides will give $x=12$. They are completely at a loss when confronted by an equation like $c-12\frac{1}{4}=10\frac{1}{2}$ since they cannot see immediately what the answer is, and they have no way of solving it. It is just at this point that the principle stated above, that this equation means that c is $12\frac{1}{4}$ greater than $10\frac{1}{2}$, is of value. If x is $12\frac{1}{4}$ greater than $10\frac{1}{2}$, it is obvious that $12\frac{1}{4}$ must be added to $10\frac{1}{2}$ in order to get the value of x. This is the fundamental way of thinking, and the axiom is

simply another and more concise expression of this principle. Expressing the left hand side of the equation as so much "short" and illustrating it on the balance or see-saw may help some pupils in seeing the relationship. In any case it will be a difficult exercise for many of the pupils.

d. *Solving equations by multiplication.* If the pupil sees that in the equation,

$$\tfrac{1}{2}n=10,$$

n is twice 10, then he will be able to solve any problem of this nature and will not make the mistake so often made of dividing 10 by 2 and getting 5 for the answer instead of 20. The axiom may be given as a shorter way of expressing it, but it is essential for the pupil to see that $2 \times \tfrac{1}{2}n$ is really equal to n.

Test I should be given at this point. It will be on solving the four types of equations discussed above. Stress will be put on solving many simple equations, but also some hard ones will be included to see if the pupils can use the principles or axioms. Checking discussed below will also be expected. No equations involving more than one rule will be given in this test. After the test the papers should be checked carefully by the teacher to see what remedial work will be necessary. Be sure the pupils understand this part of the unit before taking up the next section. Extra problems will be found in Section V for those who are slow in attaining the required skills, and for those who have to wait awhile to go on. An attempt is made to set the problems for the later group on a higher and also interesting plane so that they will enjoy the work and be benefited by it. This will be discussed more fully in C below.

5. *Checking results.* The pupil should early form the habit of checking the results of all equations he solves. This is to be done by substituting the answers obtained into the original equations. He will be expected to know this to solve the problems on the tests.

6. *Problems involving the use of equations.* Except for illustrative and motivating purposes few problems should be given until the pupil has mastered the solution of simple equations. To be able to solve problems that involve the use of

equations it will be necessary for the pupil (1) to be able to use the language of algebra, that is translate the parts of the problem into algebraic expressions, (2) to be able to form an equation from these expressions, (3) to solve the equation, and (4) to interpret the results and obtain the answer to the problem. Of these four things, No. 3 should have been automatized by this stage; the others should be taught carefully. The answer to the equation will usually be the answer to the problem, but the pupil should understand the meaning and be able to check his work by putting the answer back in the original problem. Sometimes it will be necessary to subtract (or add) the answer to the equation from (or to) some other number in order to get the answer to the problem. The importance of this method of solving problems may be brought out by having the pupils solve some problems by both the arithmetical and the algebraic methods. A number of problems should be given for practice in solving by the use of equations. A list will be found in Section V, if the list in the text is not sufficient.

7. *The use of more than one law to solve an equation.* Too often text book writers and teachers assume that if a pupil can use the separate rules or axioms of equations, he should be able to solve an equation in which it is necessary to use two or more of the rules. Definite attention should be given to this phase, for usually most of the pupils have some trouble with it. A test should be given by the teacher before he assumes that all the group can solve equations in which more than one rule is needed. This test need not be long but should be so constructed that it reveals just what type of teaching is still necessary. More difficult problems may be given after the pupils have learned to use two or more rules on one equation.

Test II should be given as the final test of the unit. It covers the solving of the equations needing one rule, or several, and problems of the type used throughout the unit as well as the ability to check answers.

C. Use of the supplementary drill exercises and problems. (For a discussion of C see Teaching Plan No. 1.)

Section VII. CORRELATIONS

A. Wth other subjects.
   1. *Shop courses* of any sort may contain formulas. The work on equations should help in evaluating these for any particular term.
   2. *With science.* Formulas appear in the eourse in science, but not to any great extent in that of the seventh and eighth grades. Equations should help in understanding those which do appear.

B. Within the subject.
   1. *With measurement* by similar triangles and by the use of numerical trigonometry the equation is very useful; these units should follow shortly after the work on equations.
   2. *With ninth grade algebra.* It is not necessary to say anything about the conneetion between the equation and the algebra of the ninth grade. It may be said, however, that it is important that the work on equations in grade eight be so taught that the pupil will not have to unlearn anything in order to go on with the algebra of grade nine.

C. Within the unit.
   1. *The relation between* the addition, subtraction, multiplication and division axioms.
   2. *The relation between* the solving of simple equations and the solving of those which require the use of more than one axiom.

Section VIII. BIBLIOGRAPHY

A. For teachers.

Everett, J. P. The Fundamental Skill of Algebra. Teachers College Bureau of Publications, 1928. pp. 84-100.

A good discussion dealing with the nature of equations and suggesting a different way of teaching and solving them.

Schorling, R. and Clark, J. R. Modern Mathematics, Eighth School Year. World Book Company, 1929. Chapter III.

A good treatment of the topic of equations with drawings of the balance to show the meaning of the laws of equations or the axioms.

Smith, D. E. The Progress of Algebra. Ginn and Company, 1925. pp. 7-13.

A discussion of the use and value and nature of equations and formulas.

Smith, D. E. and Reeve, W. D. The teaching of Junior High School Mathematics. Ginn and Company, 1927. pp. 173-179, 302-308.

A discussion of the place and use of the formula and the equation in elementary algebra. Also a model lesson on the development of the axioms.

Stone, J. C. The New Mathematics, Book II. Benj. H. Sanborn & Company, 1926. Chapter II.

A careful introduction to equations, containing a list of over 60 practical problems at the end.

Strayer, G. D. and Upton, C. B. Junior Mathematics, Book II. American Book Company, 1929. Chapter III.

This book contains a well graded unit on the equation for the first part of the eighth year.

B. For pupils.

Schorling and Clark, Op. cit. 31-50.

Stone, J. C. Op. cit. Chapter II.

Strayer and Upton, Op. cit. Chapter III.

Section IX. TESTING PROGRAM

Prestest

I. Place the answers to the following equations to the right:

1.	$x + 7 = 10$	( )	5.	$3x + 24 = 48$	( )
2.	$3n=9$	( )	6.	$5n=45.5$	( )
3.	$m - 2 = 10$	( )	7.	$2m - 3.6 = 4.8$	( )
4.	$\frac{1}{2}x=5$	( )	8.	$x/5 = 10$	( )

II. Check each of the following to determine if correct. If correct, mark with (C). If wrong mark with (W).

1.	$x+8=18$ answer 10	( )	3.	$2x-4=16$ answer 6	( )
2.	$3a = 15$ answer 6	( )	4.	$\frac{1}{3}x=2$ answer 6	( )

III. Albert sold a dog for $18, which was $7 less than the dog cost. How much did the dog cost? (Work by using equations.)

Test I.

Time 20 minutes

I. Place the answer of each equation to the right.

1.	$x + 5 = 10$	( )	6.	$7n = 42$	( )
2.	$4p = 8$	( )	7.	$n - 2 = 43$	( )
3.	$x - 4 = 6$	( )	8.	$c/5 = 85$	( )
4.	$\frac{1}{4}x = 3$	( )	9.	$x + 256 = 524$	( )
5.	$n + 7 = 3$	( )	10.	$32x = 192$	( )

11.	$b - 217 = 186$	( )	14..	$115x = 6.90$	( )
12.	$\frac{1}{3}x = 47$	( )	15.	$x - 2.5 = 2$	( )
13.	$n + 41.1 = 89.2$	( )	16.	$1/10x = 10\frac{1}{2}$	( )

II. Check each of the following to determine if correct.
If correct mark with (C). If wrong, mark with (W).

1.	$x - 7 = 21$	answer 15	( )
2.	$4n = 32$	answer 8	( )
3.	$p - 8 = 17$	answer 9	( )
4.	$\frac{1}{2}x = 18$	answer 9	( )
5.	$x - 236 = 442$	answer 206	( )
6.	$3p = 27$	answer .9	( )
7.	$h - 3.2 = 8$	answer 8	( )
8.	$x/3 = 8$	answer 24	( )

Test II. Unit Achievement Test (54:7)

I. Write an equation for the following problem.
If John had $12 more, he would have a total of $35.00.
How much money does he have? (_____)
(Score one point for correct answer.)

II. Solve the following equations and place the answer at the right.

1.	$y + 8 = 15$	( )	11.	$12 = 5x - 3$	( )
3.	$3x = 18$	( )	12.	$3\frac{1}{2}t = 7$	( )
3.	$x - 9 = 14$	( )	13.	$x/5 = 6/3$	( )
4.	$n/3 = 20$	( )	14.	$\frac{2}{3}x = 18$	( )
5.	$5x + 7 = 17$	( )	15.	$5x + 3x + x = 36$	( )
6.	$3m + 2m = 20$	( )	16.	$3/x = 6$	( )
7.	$z/4 = 12\frac{1}{2}$	( )	17.	$8 = x/4$	( )
8.	$1/5x = 35$	( )	18.	$15 = y - 8.4$	( )
9.	$16 + 3x = 34$	( )	19.	$9x - x = 40$	( )
10.	$y/5 = 4$	( )			

(Score one point for each correct answer.)

III. Check each of the following answers to determine if correct.
If correct, mark with (C). If wrong, mark with (W).

1.	$x+5=9$ ans. 4	( )	6.	$3x+2x=20$ ans. 4	( )
2.	$x-6=12$ ans. 6	( )	7.	$7x-7=16$ ans. 2	( )
3.	$14=n+6$ ans. 20	( )	8.	$1\frac{1}{2}n=9$ ans. 6	( )
4.	$15=x-14$ ans. 29	( )	9.	$x/5 = 10$ ans. 2	( )
5.	$x-1\frac{1}{2}=3\frac{1}{2}$ ans. 5	( )	10.	$10/y=20$ ans. 2.	( )

(Score one point for each correct answer.)

Total maximum score — 30 points.

TEACHING PLAN NO. III.

Section I. IDENTIFICATION

Teacher: _____ Grade: 8 Date: _____
School _____ Subject: *Mathematics*
Title: *Practical Indirect Measurements* Duration: *3 weeks*

Section II SUBJECT MATTER OUTLINE

Practical Indirect Measurements

A. Congruent triangles
   1. Making exact copies of triangles
      a. using the angle, side, angle relation
      b. using the side, angle, side relation
      c. using the three sides
   2. Measuring distances by the use of congruent triangles

B. Similar triangles
   1. Introduction
   2. Definition
   3. Equality of angles
   4. Equality of ratio
   5. Other similar triangle constructions
   6. Problems

C. Scale drawing
   1. The meaning of scale drawing
   2. The measurement of distances by the use of scale drawing in right triangles
   3. The measurement of distances by the use of scale drawing in other kinds of triangles
      a. the side, angle, side triangle
      b. the angle, side, angle triangle.

Outline for Supplementary Work

A. For remedial purposes
   1. Problems on congruent triangles
   2. Problems on similar triangles
   3. Problems on scale drawing

B. For the better group of pupils
   1. Problems on congruent triangles
      a. the three side congruence theorem
      b. the proof of the equality of lines and angles by means of the congruence theorems (use demonstration)
      c. the solution of harder problems by the use of the congruence theorems

2. Problems on similar triangles
   a. harder problems dealing with similar triangles
   b. other constructions of similar triangles
3. Problems on scale drawing
   a. harder problems dealing with scale drawing
   b. other uses of scale drawing

Section III. Pupil Inventory
(See Section III Teaching Plan No. I.)

Section IV. Aims or Objectives

A. General aims
   1. To give the pupil some of the simple concepts of geometry
   2. To show the pupil the value and appreciation of the art of indirect measurement in the actual world
   3. To give the pupil a deeper appreciation of the value of mathematics in doing the work of the world

B. Specific aims
   1. Items of knowledge
      a. concepts
         congruence in two cases, angle, side, angle and side, angle, side
         indirect measurement
         similarity
         scale drawing
         relation between congruence and similarity
         relation between congruence and similarity and scale drawing
         land measurement
      b. need and uses of these concepts
         need for land measurements
         use of congruence, similarity and scale drawing concepts in making indirect measurements
      c. the meaning and spelling of the following words and significant phrases
         angles
         included side
         congruent
         included angle
         degree
         unknown distance
         measuring indirectly
         similar triangles
         equations
         ratio of sides

corresponding angles
corresponding sides
to construct triangles
right triangles
scale drawing
draw a triangle to scale

2. Skills
   To develop the following abilities
   a. the ability to measure angles and lines necessary to construct congruent triangles
   b. the ability to use these measurements to make copies of triangles by two rules, angles, side, angle and side, angle, side
   c. the ability to use the above skills in finding unknown distances indirectly
   d. the ability to measure the distances and angles needed in constructing one triangle similar to another
   e. the ability to write the ratio of the corresponding sides of similar triangles
   f. the ability to solve the equation formed by equal ratios
   g. the ability to use d, e and f above in finding actual unknown distances
   h. the ability to read and understand fully a scale drawing
   i. the ability to make a complete scale drawing having given the data
   j. the ability to make the measurements needed in finding distances by the scale drawing method
   k. the ability to use the data of j in finding actual unknown distances
3. Attitudes and appreciations
   a. to appreciate the methods used by surveyors in obtaining the measurements of inaccessible distances
   b. an appreciation of the use that is made of the principles of geometry in this type of work
   c. an appreciation of the value of precision in making measurements
   d. an appreciation of the importance of checking some work of this sort, as in bridge building and the like
   e. an appreciation of the things which a good engineer and surveyor must know in order to do his work properly

Section V. Activities and Exercises

A. Activities.

1. Measure ground distances by means of congruent triangles. Measure some that can be checked.
2. Make some measurements using farm level or transit if one is available.
3. The same may be done for scale drawing and for similar triangles.
4. Examine blue prints of buildings or bridges if such can be found, these are really scale drawings. Concentrate on the simpler of the blue prints.
5. Examine house plans or make the floor plan for the school building.

B. Exercises.

1. For use as regular class work
   (Exercises for regular teaching work will be found in the basic text (38) on pages given in the course of study. The supplementary texts by Strayer and Upton (48) and Knight, Studebaker and Ruch (18) may be used when needed.)
2. For use as supplementary exercises to care for individual differences
   a. for remedial purposes
   (the problems listed below are of about the same difficulty as those appearing in the unit.)

Problems on congruent triangles

Strayer and Upton, Book Two.# pp. 223-226.
Stone, Book II. pp. 75-77.

Problems on similar triangles

Knight, Studebaker, Ruch, Book Eight. pp. 389-390, problems 1-9.
Strayer and Upton, Book Two. pp. 241-248.

Problems on scale drawing

Strayer and Upton, Book Two. pp. 249, problems 1-6.

1. Measure the dimension of this page to the nearest $\frac{1}{2}$ inch, and make a drawing of the page to the scale of $\frac{1}{4}$.
2. Measure the top of your desk to the nearest inch, and draw a plan to the scale $\frac{1}{8}$.
3. If a line 1 inch long in a plan represents a distance of 8 feet, what distance is represented by a line 3 inches long? by a line 4 inches long? by a line $1\frac{1}{2}$ inches long?

The complete title of these books will be found in Section VIII, Bibliography.

4. If a scale is 1 inch to 10 feet, what length line on the map will represent 5 feet on the ground?
5. A drawing of a rectangular school garden is drawn to the scale of 1 inch to 2 feet. The plan is 18 inches long and 12 inches wide. What are the dimensions of the garden?

b. for the better group of pupils

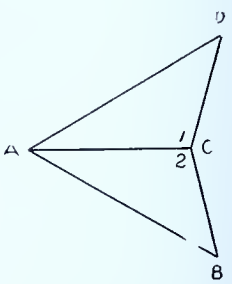
On congruent triangles

1 The three side congruence theorem.
Strayer and Upton, Book Two. p. 223.

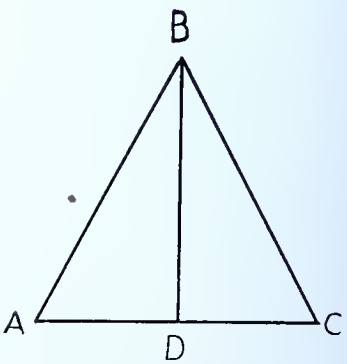
2. Proving lines and angles equal by means of congruent triangles

(As soon as the congruence theorems are accepted, the pupils will be able to demonstrate that lines are equal or that angles are equal. It is possible to do this with 8th grade pupils, (34) though probably the better part of the class will profit most by it. Before they can understand measurements it will be necessary for them to know the meaning of corresponding parts. When this meaning is acquired the pupils will be able to demonstrate problems like those listed below.)

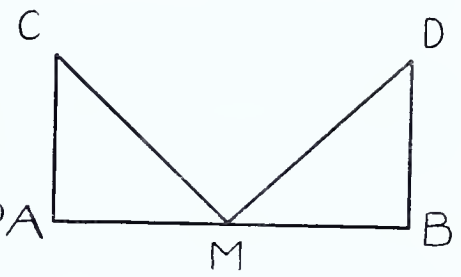
1. Given: AC bisects angle A
and angle 1=angle 2
To prove: DC=BC



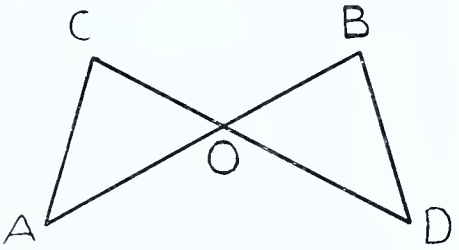
2. Given BD perpendicular to AC
and BD bisects angle B.
To prove: AB=BC



3. Given:
CA perpendicular to AB,
DB perpendicular to AB,
AC=BD, and that M is the mid-point of AB.
To prove: angle C=angle D



4. Given: AB and CD are straight lines, AO=OD, and
angle A = angle D.
To prove:
angle C = angle B.



Harder problems on similar triangles

1. If a telephone pole casts a shadow 28 feet long at the same time that a boy 5 feet high casts a shadow 4 feet long, how high is the pole? Draw a figure to scale.
2. A tree casts a shadow 56 feet long at the same time that a post 8 feet high casts a shadow 14 feet long. How high is the tree?
3. A water tower casts a shadow 87 feet long at the same time that a baseball bat made to stand upright casts a shadow twice its length on a level sidewalk. Find the height of the tower.
4. The length of a leaf is 2 inches and its width is 1 inch. After the leaf has grown ½ inch longer, keeping the same shape, what is its width?

(Problems like these may be found in many geometries. Figures are usually included.)

Other ways of constructing similar triangles

The method of constructing similar triangles explained in Schorling and Clark, Book II may be also used for problems dealing with photography. See also Wentworth, Smith, Brown. Book I, p. 211. The following problems from the latter book are suggestive.

1. A man 5 feet 8 inches tall stands 16 feet from a camera, which is 8 inches long. What will be the height of his photograph?

2. A photograph of a man who is 5 feet 8 inches tall is 4 inches high and the camera is 10 inches long. How far did the man stand from the camera?
3. A tree photographed by a 4 inch camera 25 feet away is 3 inches high in the picture. How high is the tree?
4. If a boy's face is 8 feet from the camera which is 10 inches long, what is the ratio of the length of the photograph of his face to the length of his face? If he moves his hand 2 feet nearer the camera, what is the ratio of the length of the photograph of his hand to the length of his hand?

(Problems dealing with the pantagraph, explained in most geometries, are also very interesting.)

On Scale Drawing

1. To find the height of a tall church spire, Walter measured the angle between the horizontal line and the line of sight and found it to be 60°. He then walked 100 feet nearer and found that an angle measured in the same way was 80°. Draw a plan to some selected scale and find the height of the spire.
2. Frank rode from A to B, 6 miles; then from B to C, 8 miles. At B the angle ABC was 120°. If he could ride directly back in a straight line from C to A, how many miles could he save?
3. When a distance of 150 miles is represented on a map by 1½ inches, what distance is represented by 3¼ inches.
4. When a distance of 200 miles is represented by 1½ inches, what length represented 500 miles?
5. The angle of elevation of the top of a tree is 30° when observed from a point 40 feet from the foot of the tree. How high is the tree above the horizontal line of sight?

Section VI. Teacher Procedure and Means

Outline

A. The use of the teaching plan during teaching
B. Detailed procedure in teaching the unit
C. The use of the supplementary drill exercises and problems

A. The use of the teaching unit during teaching

(For a discussion of A see Teaching Plan No. I

B. Detailed procedure in teaching the unit.

1. *Introducing the unit.* This unit should be introduced in such a way that the children understand and see the need for making indirect measurements. This may be done in various ways and should depend on the conditions in the com-

munity. For example, if some public survey is being nearby, it will be an excellent opportunity for pupils to see the principles in actual use. On the other hand, some altogether different example might be taken in another community. A few suggestions of general nature for such introductions will be given here. The teacher should not fail to take advantage of any special activity that may arise in the community.

In most instances pupils will be familiar with some sort of land marks, as lot corners or section corners. They will have seen some surveying on streets or roads or farm leveling or the like. There should be enough of this to enable them to have a general idea about a surveyor's transit and possibly other tools. Very good pictures of these appear in many textbooks and in catalogues for scientific apparatus. By a discussion of the need of knowing where one's land is the pupils can see the need of measurements. Most of them will know something about the way land is divided and perhaps of disputes over the ownership of land, on account of inaccurate measurement years before.

The purpose so far has been to show the meaning and use of measurement in general. The next thing will be to show the need of measuring distances that cannot be measured directly. Show how the surveyor must sometime run a line through a small swamp, or lake of water, or measure the heights of cliffs or houses, or even trees. Attention could be called to the distances the pupils have learned about in geography which could not have been measured directly. All of them will know something about the distances to the sun, moon, and stars, which have been measured indirectly.

Another way of introducing the topic is to deal directly and solely with measurements which appeal to children. Many of the boy scouts have made measurements as Napoleon's soldier is supposed to have done, that is, by standing on the river bank and holding his head in such a position that a point on the opposite bank was just in line with the brim of his hat; then by facing to the right or left and without changing the position of his head, he locates a corresponding point on the bank of

the river. He is then able to measure this. This will usually appeal to the pupil, and such measurements may actually be made as a preliminary activity.

2. *Measuring by the use of congruent triangles.* a. *Teaching the construction or intuitive proof of the two congruence theorems.* The project discussed above in connection with the scout's measurements will lead directly into congruent triangles and may well be used in connection with any sort of introduction of the unit. The pupil may be made to see that triangles are exactly equal, and that they should be made to want to know the conditions for this congruence. For the purpose of indirect measurement only two theorems of congruence are needed, the side, angle, side theorem and the angle, side, angle theorem. The intuitive proof is probably best presented from the standpoint of measurement, and the following class room procedure is suggested. This tallies closely with the procedure outlined in the Schorling and Clark book. First of all, have all the pupils draw a triangle on their paper according to given measurements, as side AB=12 cm., angle A=40°, etc. (Metric measurements are used here because the children cannot make a triangle with an 8 in. side on their papers.) After they have finished and cut out their triangles, comparisons may be made, first, with the pupils sitting next, and finally let the teacher collect them and compare the whole lot. Some of course will not fit, and the children themselves will probably suggest that the measurements may not have been made carefully; have these checked for accuracy. After this have each child draw any triangle on his paper and then measure two sides and the included angle. With these data, which will be different for each pupil, have the pupils construct and cut out a triangle as before. They will compare this with the original triangle on their paper. Again have those whose triangles fail to fit check their work. After they have done this, they are usually willing to admit that the condition used is sufficient to make two triangles congruent. Have them make their own statement of this fact and be sure that all understand the meaning. Afterward the more elegant form of the statement of the theorem may be given. The second congruence theorem may be proved, intuitively, after the same fashion.

b. *Teaching the use of these theorems to measure inaccessible distances.* (1) Three cases are considered in the use of these theorems; the first will be the use of the first theorem, the side, angle, side. After this first theorem has been proved and a few illustrations given on the board, it would be well to let the class actually measure some distances out on the school ground. Some of these should be distances that could be checked in order that the pupils may gain confidence in their work. After a few experiences of this sort they may well afford to come back to the class room and spend some time in drill on problems depicting such situations. When they are thoroughly familiar with the method, more outdoor work may be done.

It is important here that the teacher have in mind the fact that this theorem will in nearly all practical cases involve vertical angles as the angle included between the correspondingly equal sides in each triangle. This theorem has usually been presented in some earlier unit. If it happens that the pupil has not met it before, it is suggested here that the pupil be allowed to assume it without any sort of proof, provided that satisfies him, as it will in most cases.

(2) The second case will deal with problems in which use will be made of the other theorem, angle, side, angle. The procedure will be about the same. Afterwards measurements should be made in both ways on the same field trip, and the pupils should be led to see the essential difference in the two methods and the advantage of each. It should be noted here that the pupils will have to construct angles equal to given angles. If the work is being done with a farm level or transit, it will be easy to get the angles equal. If neither of these is available, the difficulty may be met in one of two ways. The angles may be constructed by the use of the compasses and straight edge principle, using a line of sight for a straight edge and a string or rope for a compass. Or the pupils or teacher may construct a field protractor and use it to get the angles equal. This is not hard to do and would be an excellent activity to be carried on in connection with the class work. If such an instrument has been made in some earlier unit it could be used here.

(3) It will probably be wise to confine the above work to cases of measurement on the ground. The third case then will be different only in that it will deal largely or wholly with vertical measurements. To make these measurements corresponding triangles must be drawn on the ground. The work for the whole group should probably be confined to measuring the heights, so that the angle at the ground may be considered a right angle. Other and more difficult problems will be considered for the better part of the class.

At this point the teacher should give Test I (Section IX). It should tell the teacher just where the pupil is weak. The results should be used to determine whether the class as a whole needs extra work on parts of the unit, or whether most of them have what was required. It is probable that the individuals will be weak at certain points. These things should be made the subject of reteaching. If some of the class has to wait while this is being done, they can be profitably employed in working some of the supplementary problems for better pupils listed in Section V. The test is made to cover the terms used and the desired skills for this section of the unit. If the teacher feels that it is desirable to add other questions, it will be quite proper to do so. The time limit is purely arbitrary.

3. *Measuring by the use of similar triangles.* a. *Teaching the meaning and definition of similar triangles.* The exercises on page 55 in the Schorling and Clark book, form an excellent introduction to the meaning of similar triangles. The only object in mind here is to show that if two triangles have two angles of one equal to two angles of the other, the triangles are similar, or merely that they have the same shape. The conclusion is strictly intuitive, and no attempt should be made to make it otherwise at this point. The pupils are now ready for the definition of similar triangles as triangles that have the same shape, and for the statement of the principle that triangles are similar if two angles of one are equal respectively to two angles of the other. More construction problems can be given here to advantage probably. They can easily be devised by the teacher as mere drill problems to clinch the idea.

b. *Teaching the relation between the sides of similar triangles.* The relation between the sides of similar triangles

is the important thing in this part of the unit. It is probably best done by careful measurement as described in the textbook. The best plan is to use the centimeter side of the ruler. After a review of proportion the pupil can state the principle of corresponding sides.

c. *Teaching the measurements of distances by the use of similar triangles.* A review and some extra drill on equations will put the pupil in position to solve simple problems of this sort. The shadow reckoning problems are the simplest of this sort since they deal with the right triangle. Problems dealing with oblique triangles may also be given. At this point the pupil is ready to do actual outdoor measurements using these principles. This should be done if at all possible. It will do more to give the pupils a clear idea of the whole and an appreciation of its value than anything else. If any sort of surveying instruments are available, it will add much to the interest of the class to explain and use them. Simple instruments may be made as extra work by the better members of the class, or as a class project. (41:195-223)

d. *Teaching the other method of constructing similar* triangles. The other method of constructing similar triangles involves the principle, that if two corresponding pairs of sides of two triangles have the same proportion and the included angles are equal, the triangles are similar. The pupils will probably accept the truth of this principle without question. The actual drawing of the triangles may be done according to either of the two ways used in the text-book by Schorling and Clark, depending on the way the problem is described. It must be kept in mind, however, that the included angles must be made equal. This will be taken care of in the first case by the fact that they are vertical angles, but they need to be so constructed in the second case.

A slight variation of these triangles will serve as an excellent exercise for the better members of the class. In making a picture with a camera, the image on the film is upside down, but triangles formed by the object taken and the image on the film with the rays of light that cross at the lens are similar in the same way as the ones just studied about. If one uses

the fact that the altitudes are in proportion to the corresponding sides, he may find the height of the object if he knows the size of the picture, the length of the camera, and the distance from the object to the camera.

Test II should be given at this point when the work on similar triangles is finished. Follow the same directions about reteaching. The time limit here is also arbitrary.

3. *Measuring by the use of scale drawing.* a. *Teaching the meaning of scale drawing.* (1) *Explanation with house plans, blue prints and the like.* Two suggestions will be made as to the way of introducing and giving the meaning of scale drawing. The first will be by the use of the scale drawing that is used in house plans and all sorts of blue prints. It is probable that the children have seen some sort of blue prints, at least at a distance, and will be easily interested in knowing more about them. It is advisable, probably to begin by having the pupils make a drawing of their garage, school room, or their room at home, or some such simple plan. Have them make the scale one inch to one foot if the work is done on the blackboard, or one centimeter to one foot if the work is done on paper. This will be a simple problem but will involve all the elements of a scale drawing. They can see how the distances could be found, if they had the plan and the scale. If blue prints of any sort can be found, they can be examined with profit. Many times house plans may be obtained from books or magazines which will answer the purpose. Maps may also be used.

It is also possible to use a squared paper in introducing the subject. Present some practical problem to the pupil, like measuring the diagonal distance across the room, and then suggest that he draw a figure of the room on squared paper or on a squared blackboard if one is available, as one should be in every mathematics class room in the country. By setting a convenient scale the pupil can see that if the diagonal is measured and allowed to represent a corresponding scale, the length of the diagonal of the room can be easily found. It would probably be best to use a one to one correspondence here also in the beginning.

(2) *General meaning.* From this beginning it is possible to derive the general meaning of scale drawing. The general meaning is that in the drawing constructed the parts bear a definite proportion to the original figure; this is true for all the lines of the figure. The pupils must realize that it is necessary to construct certain parts so that they will bear this proportion, and they must know what parts are necessary.

b. *Relation to the congruent method.* It is very much worth while that the pupil see the relation between the method of scale drawing and the method of congruence already studied. In the method of congruence enough measurements are made so that the triangle may be reconstructed in such a place that the desired length or angle will be accessible to measurement. This requires quite a good sized area in some cases, and of course in some cases it would be very inconvenient if not impossible. Scale drawing is introduced for the purpose of avoiding this difficulty and making the work more convenient. The same measurements are neeessary but the construction may be done on a paper rather than on the ground; otherwise the work is the same. In other words the method of congruent triangles uses a one to one correspondence, and scale drawing uses a one to x correspondence, where x represents the scale of the drawing.

c. *Teaching the measurement of distances in right triangles.* The measurement of distances by scale drawing involves three things: 1. Measuring the proper lines and angles on the ground figure; 2. Constructing the angles and constructing the lines to the proper scale to make the scale figure; 3. Measuring the unknown line of the scale figure and determining what distance the line represents. It is essential that the pupils know how to do each one of these, and the teacher should consider each in his teaching. If skill is lacking in one particular, the pupil will fail to get the desired distance.

Measurement with right triangles is simpler since it is necessary to make a fewer number of measurements on the actual ground figure. The scale figure is also easier to make; this is especially true if the pupil uses a squared paper on

which to make his drawings. The right triangle can be drawn directly on the paper.

d. *Teaching the measurement of distances in other triangles.* What has been said above in regard to right triangles applies in large measure to other triangles also. The problem is to construct a triangle like the original one to a chosen scale, and compute the unknown distances. Two types of triangles are available for use, those in which the pupil can measure two sides and the included angle, and wishes to find the third side, and those in which he can measure two angles and the included side and wishes to find either of the other two sides. Work with these triangles will be largely with the better pupils, probably.

Inspection Test, Geometry IV, page 14, of the Course of Study Bulletin, (54) should be used as a final test when the unit is completed.

C. The use of the supplementary drill exercises and problems. (For a discussion of C see Teaching Plan No. 1.)

Section VII. CORRELATIONS

A. With other subjects.

1. *With geography or social science.*
   a. Scale drawing correlates well with the study of maps in geography. Maps are merely scale drawings. The work in the mathematics class should give the pupil a much clearer idea of the ways maps are made and the way in which they may be used. Small maps may be made of the school grounds or some plot near by. (41)
   b. The indirect measurements referred to in geography, for example, the distance to the sun and the moon and the like, are more easily understood by recalling the way indirect measurements are made in mathematics.
2. *With general science.* Many indirect measurements are referred to in the study of general science, for example solar distances; these may be more easily understood by the principles of mathematics.

B. Within the subject.
   1. *With demonstrative geometry.*
      a. Congruent triangles will also be studied in plane geometry, but usually from a different standpoint. However, the terms and theorems will be the same. Applications are identical also.
      b. For similar triangles the same is true.
   2. *With trigonometry.* The work on similar triangles leads directly to the fundamental relations of trigonometry.

C. Within the unit.
   1. *The relation between* similar triangles and congruent triangles.
   2. *The relation between* congruence and scale drawing.
   3. *The relation between* scale drawing and similarity.

Section VIII. BIBLIOGRAPHY

A. For teachers.

Breslich, E. R. Junior Mathematics, Book I. The Macmillan Company, 1925. Chapter V.
A good discussion of measurement in general with uses which appeal to the junior high school pupil.

Knight, F. B. Studebaker, J. W. and Rueh, G. M. Standard Service Arithmetics, Book Eight. Scott, Foresman Company, 1928.
This book contains a good introduction to similar triangles, with applications.

National Council of Teachers of Mathematics, Third Yearbook. Teachers College Bureau of Publications, 1928.
This book contains many articles on the teaching of mathematics. The making of simple instruments is described on pages 195-222.

Stone, J. C. The New Mathematics, Book II. Benj. H. Sanborn Company, 1926. Chapter IV.
This is a good chapter on the measurement of inaccessible distances by all three methods. It contains many problems.

Strayer, G. D. and Upton, C. B. Junior Mathematics, Book III. American Book Company, 1929. Chapters XIII and XV.
A well graded discussion of congruence and similar triangles, with many practical problems.

Wentworth, Geo., Smith, D. E. and Brown, J. C. Junior High School Mathematics, Book I. Ginn and Company, 1925. pp. 203-216.

A discussion of similarity and scale drawing with optional topics and many interesting problems.

B. For pupils.

Breslich, E. R. Op. cit. Chapter V.
Knight, Studebaker, and Ruch. Op. cit. pp. 381-389.
Stone, J. C. Op. cit. Chapter IV.
Strayer and Upton. Op. cit. Chapters XIII and XV.
Wentworth, Smith, and Brown. Op. cit. pp. 203-216.

Section IX. Testing Program

Test I.

Time 20 minutes.

1. The side between two angles of a triangle is called the _____side.
2. When two triangles fit exactly, they are said to be_____.
3. An angle is measured in_____.
4. The angle between two sides of a triangle is called the _____angle.
5. It is possible to find the length of some lines even though we cannot get to them to measure them directly.
   Yes_____. No_____.
6. Draw any kind of a triangle in the space to the right. With your ruler and protractor measure two sides and the included angle and write the sizes below.
   Side_____, Angle_____, Side_____.
   Write the measurements on the triangle.
7. Use the measurements just found and make a triangle congruent to the triangle of Ex. 6.
8. Draw a figure to show how you would measure the width of a pond which is too wide to permit stretching a tape across.

Test II.

Time 20 minutes.

1. Triangles that have the same shape are_____.
2. Two pairs of corresponding sides of similar triangles have the same_____.
3. Angles that are in the same position in similar triangles are called_____angles.
4. In similar triangles the corresponding angles are_____.

5. Draw a triangle on this paper and measure the angles necessary to have in constructing a similar triangle. Put the results on the triangle and also below.

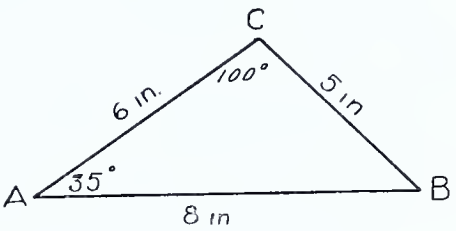
_____

6. Use these results to construct a triangle similar to the one in Ex. 5.
7. Measure two pairs of corresponding sides of the similar triangles just drawn and put the results on the figures.
8. Suppose it was impossible to measure the third side of the second figure. Show how you could find the length.
9. When a fence post 6 feet high casts a shadow 4 feet long, how high is a water tower whose shadow, at the same time, is 60 feet long? Draw a figure and do the work on this page.

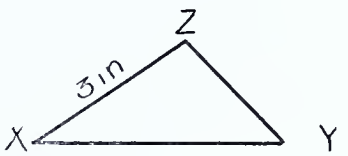
Test III. Unit Achievement Test (54:14)

Time 20 minutes.

1. To make an exact copy of a triangle it is necessary to measure either two_____ and the_____ _____ or two angles and the_____ _____ of the given triangle.
(Score 1 point if all the blanks are filled correctly.)
2. Similar triangles always have their corresponding_____ _____equal. (Score 1 point.)
3. Trianges ABC and XYZ are given similar.



1′ Angle X = _____
2′ Angle Y = _____
3′ XY = _____
4′ YZ = _____



(Score 1 point each, total score 4 points)

4. If a boy 5 feet high casts a shadow 6 feet long, a flag pole that casts a shadow 120 feet long is_____high.
(Score 2 points)
5. Underscore the word that best completes the following statement. A scale drawing and the figure it represents

have (sides, angles) of the same size.
(Score 1 point)
(Total score 9 points)

TEACHING PLAN NO. IV.

Section I. IDENTIFICATION

Teacher: _____ Grade: *9-A*. Date: _____
School: _____ Subject: *Mathematics*
Title: *Square root, Radicals and Exponents.* Duration: *3 weeks*

Section II. SUBJECT MATTER OUTLINE

Square root, Radicals, and Exponents

A. Square root
   1. The need for square root
   2. The meaning of square root
      a. square root and equations
      b. graphic device for finding squares and square root
   3. The square root of an agebraic expression
   4. The square root of an arithmetical number
   5. The new axioms used in square root
   6. The application of square root to the law of Pythagoras
   7. The geometric representation of the square root of an arithemtical number
   8. The square root of a product
   9. The square root of a fraction

B. Radicals
   1. The definition of a radical
   2. The simplification of radicals
   3. The addition and subtraction of radicals

C. Exponents
   1. The definition of exponents
   2. The exponent and the index number of a radical
   3. The fractional exponent
      a. meaning
      b. laws
   4. The zero exponent
      a. meaning
   5. The negative exponent
      a. meaning

Outline for Supplementary Work

A. For remedial purposes
   1. Square root
      a. of arithmetical numbers
      b. of algebraic expressions
      c. applications of square root

2. Radicals
   a. the simplification of radicals
      (1) involving whole numbers
      (2) involving fractions
   b. the addition and subtraction of radicals
3. Exponents
   a. the evaluation of expressions involving fractional exponents
   b. the evaluation of expressions involving zero and negative exponents

B. For the better group of pupils
1. Square root
   a. of algebraic expressions
   b. harder problems involving square root
2. Radicals
   a. harder problems involving the simplification of radicals
   b. the fundamental operations with radicals
      (1) addition and subtraction
      (2) multiplication and division
3. Exponents
   a. harder problems dealing with fractional, zero, and negative exponents

Section III. Pupil Inventory
(See Section III Teaching Plan No. I.)

Section IV. Aims or Objectives

A. General aims
1. To help give the pupils the fundamental formal skills of algebra
2. To help complete the pupil's knowledge of algebra
3. To help show the pupils that all situations dealing with numbers are governed by definite laws.

B. Specific aims
1. Items of knowledge
   the meaning and spelling of the following words and significant phrases

square root	zero exponent
powers	irrational
axioms	index
law of Pythagoras	fractional exponent
radical	integral exponent
rational	negative exponent
exponent	geometrical representation

2. Skills
   To develop the following abilities
   a. the ability to find the square root of a simple algebraic expression
   b. the ability to find the square root of arithmetical numbers by the method of "division"
   c. the ability to use the two new axioms, "like powers of equal numbers are equal" and "like roots of equal numbers are equal"
   d. the ability to apply the Law of Pythagoras and to solve problems using the principle of square root
   e. the ability to find the square root of a number, one of whose factors is a perfect square and the other some small number less than 6
   f. the ability to find the square root of a fraction
   g. the ability to simplify radicals by the method of factoring and by rationalizing the denominator
   h. the ability to evaluate special radicals when the index of the root is any number
   i. the ability to evaluate simple expressions involving fractional exponents
   j. the ability to perform the operations of multiplication and division using fractional exponents and radicals
   k. the ability to evaluate simple expressions involving zero and negative exponents
3. Attitudes and appreciations
   a. an appreciation of the fact that the laws of algebra apply in all situations dealing with numbers. (57:268)
   b. an appreciation of the fact that the expressions with negative, zero, and fractional exponents are defined so that the rules of algebra apply
   c. an appreciation of the value and use of tables of squares and square roots
   d. an appreciation of the value of knowing how to find square root by the division method

Section V. Suggested Activities and Exercises

A. Activities
1. A large graphic chart (see textbook, (39:303)) may be made by individuals or as a class project. If larger squares are used with small sized divisions, the squares and square roots may be estimated more accurately.
2. If actual problems could be found in which the pupils knew the area of a figure but wanted to find a square

which would contain just this area, this would be an excellent application of square root. For example, if for some reason it would be advantageous to divide the school garden into square plats, one could divide the entire area by the number in the class and thus obtain the area each plat should be. From this data by the use of square root the side of each square could be computed. Afterward it would be possible to see if the entire garden could actually be divided into these squares.

It would also be possible to have the pupils figure just what sized space each has on the school room floor, also just what sized window space.

3. Actual unknown distances may be measured by the use of the Pythagorean relation. This would be a good class activity and would usually involve the ability to measure lines and to construct right angles in addition to the ability to solve square root.

B. Exercises

1. For use in regular class work
(The exercises found in the textbook (39) will probably be sufficient for the regular class work.)
2. For use as supplementary exercises to care for individual differences
(These problems are listed from three books which it seemed most likely that the teacher would have at hand.) (23) (55) (9)

a. for remedial purposes

(The problems listed below are of about the same difficulty as those appearing in the unit.)

On square root

1. Arithmetical numbers

Wells and Hart*	problems 1-20, p. 253.
	problems 1-12, p. 254.
Englehardt & Heartter	problems 1-27, p. 292.
Newell & Harper	problems 1-35, p. 204.

2. Algebraic expressions

Wells and Hart	problems 1-16, p. 249.
	problems 1-5 p. 251.
Englehardt & Heartter	problems 1-25, p. 287.
Newell and Harper	problems 1-28, p. 201.
	problems 1-3, p. 202.

* For complete titles see Section VIII.

3. Applications

Wells and Hart	problems 1-12, p. 263.
Englehardt and Heartter	problems 1-10, p. 293.
Newell and Harper	problems 1-10, p. 210.

On radicals

1. Simplification of radicals
   a. whole numbers

Wells and Hart	problems 1-12, p. 225.
Englehardt and Heartter	problems 1-10, p. 296.
Newell and Harper	problems 1-15, p. 206.

   b. fractions

Wells and Hart	problems 1-15, p. 256.
Englehardt and Heartter	problems 22-33, p. 292.
Newell and Harper	problems 1-15, p. 207.

2. Addition and subtraction of radicals

Wells and Hart	problems 1-10, p. 258.
Englehardt and Heartter	problems 1-10, p. 298.
Newell and Harper	problems 1-4, p. 212.

On exponents

1. Evaluation of fractional exponents

Wells and Hart	problems 1-24, p. 337.

2. Evaluation of zero and negative exponents

Wells and Hart	problems 1-18, p. 338.
Englehardt and Heartter	problems 1-24, p. 306.

b. for the better group of pupils

On square root

1. Algebraic expressions

Wells and Hart	problems 6-20, p. 251.
Newell and Harper	problems 4-15, p. 202.

2. Harder problems involving square root

Wells and Hart	problems 11-20, p. 263.
Englehardt and Heartter	problems 11-20, p. 293.
Newell and Harper	problems 11-19, p. 211.

On radicals

1. Harder problems on simplification of radicals

Wells and Hart	problems 1-15, p. 257.
Englehardt and Heartter	problems 11-38, p. 296.
Newell and Harper	problems 15-26, p. 206.

2. Fundamental operations with algebra
   a. addition and subtraction

Wells and Hart	problems 11-20, p. 258.
Englehardt and Heartter	problems 11-24, p. 300.
Newell and Harper	problems 5-14, p. 212.

b. multiplication and division

Wells and Hart	problems 1-25, p. 259.
Englehardt and Heartter	problems 1-24, p. 301.
Newell and Harper	problems 1-16, p. 213.

On exponents

1. Harder problems dealing with fractional, negative and zero exponents

Wells and Hart	problems 25-49, p. 337.
	problems 18-36, p. 338.

Section VI. Teacher Procedure and Means

Outline

A. The use of the teaching plan during teaching.
B. Detailed procedure in teaching the unit.
C. The use of the supplementary drill exercises.
A. The use of the teaching plan during teaching.
(For a discussion of A see Teaching Plan No. I.)
B. Detailed procedure in teaching the unit.

1. *Introducing the unit.* The introduction given in the textbook may be used if it can be made real to the pupils. Some activity involving the same principle may also be used, for example, those suggested in 2 of Section V. In any case, if the introduction is to be of most value, it must be real to the pupil. He must want to get the result and actually have some use for it, or feel that the result will be used by someone. This is not always possible but a definite effort should always be made to make the situation real.

2. *Square root.* a. *Teaching the meaning of square root.* It is probable that the pupils have an idea of the meaning of square root from their junior high school mathematics. If they have the fundamental idea of the meaning of square root when dealing with numbers they can usually see rather easily the application to algebraic expressions.

The work on the use of equations is merely added to give more meaning to square root and to enable the pupil to make a graphic chart for finding squares and square roots. The graphic chart should also help the pupils in getting a clear concept of the meaning of square root.

b. *Teaching the square root of a number.* The suggested method* is less exact than the regular method** but is probably more easily understood by the pupils. It is neeessary for the pupil to have a definite idea of the meaning of square root before he can use the first method. This faet constitutes one of the main advantages of this method of teaching square root. There is probably little difference in the two methods from the standpoint of speed in finding roots. (2:46)

The Course of Study (Tulsa City Schools, Tulsa, Oklahoma, 1930) required that the regular method of finding square root be taught. This is explained in Wells and Hart, (55:237) as well as in most ninth grade algebras. The teacher is cautioned to do the teaching carefully to prevent the confusion of the two methods in the minds of the pupils. Enough drill must be given to make the pupil feel sure that he is right when he works a problem either way.

c. *Teaching the square root of an algebraic expression.* The mechanism of finding square root by the regular method is much more easily seen from the use of the algebraic expressions than from the use of arithmetical numbers. The algebraic expressions given in the text are limited to trinomials whieh are perfect squares and may be solved by inspection. Harder problems can be given as supplementary work for the better part of the class. It would be hard to justify them for the entire group, however.

d. The table of roots and powers deserves speeial mention beeause of its increasing use in more extensive form in the business world. The table in the baek of the text (39:380) is simple but very useful. Wise and careful use by the teacher

* This method consists of guessing the square root of the number and dividing the number by the estimate. If the divisor and quotient are not equal, a closer estimate may be made by choosing a number half way between the divisor and quotient. This may be continued until a fair degree of accuracy is obtained.

** By the regular method of finding square root is meant the method in whieh the number, the square root of which is to be found, is divided into periods of two digits each beginning at the decimal point. The first digit on the left of the square root is then chosen by inspection as the nearest square root of the first period and the square of this digit subtracted from the first period. To the remainder annex the next period. A trial divisor is formed by doubling the part of the root already found and annexing a zero. The second digit of the root is estimated by dividing the remainder referred to above by the trial divisor. Annex the quotient to the root already found and add it to the trial divisor to make the complete divisor. The complete divisor is then multiplied by the new digit of the root and the product subtracted. If other periods remain, the process is repeated.

will show the pupil its value and help him in making use of this efficient tool when the situation presents itself later.

e. *Teaching the applications of square root.* The applications of square root are limited in a large measure to the right triangle problem using the famous Pythagorean Theorem. It is probable that this theorem has been presented somewhere in the junior high school work. If not the pupils can arrive at the truth of the theorem intuitively by the use of figures similar to the one in the book. Also the sides of various right triangles may be measured and the truth of the theorem inferred by considering the squares of the numbers which represent the lengths of the sides. The figures used should be constructed by the pupils. Only a few problems of this sort will be found in the book. Others are listed in Section V, above, and may be found in other algebras and geometries. Some outside measurements using these principles will be more meaningful to the pupils, probably. Inaccessible distances may be measured this way, either about the school ground or elsewhere. It must be remembered that the pupils will have to be able to construct the right angles in some way on the ground, or to recognize that they have a right triangle when they are dealing with upright figures, as trees and houses. They will also have to be able to measure and construct lines. These activities can be easily devised by the teacher if the time is available.

f. The new axioms* used are very important in the later parts of algebra, but have little application in this unit except in the problems just described. The pupils will probably accept them readily. It is probably worth while to introduce them here both because of their value in this type of problems (using the Pythagorean relation) and in order that they may be used in the study of quadratic equations which follow.

g. *Teaching the geometric representation of square root.* Representing the square root of numbers by lines comes directly from the Pythagorean relation. In fact it is but another application of the principle. It is of little practical value in itself, but shows the pupil that such numbers exist and may be represented by definite straight lines. It will also serve as an

* Like roots of equal numbers are equal and like powers of equal numbers are equal.

early forerunner to the field of irrational numbers. Give some practice with this but do not over-do it.

h. *Teaching the method of finding the square root of a number one of whose factors is a perfect square.* This method of finding square root will apply to a few cases only and hence is of comparatively little importance. It affords one uses of a table of square roots, but the pupil should be made to see that it is helpful only in cases where one of the factors is a perfect square. The pupils should be shown that it will *not* work in other cases.

i. *Finding the square root of a fraction* is required more often and more drill should be given on it. The pupils should be taught to change the form of the fraction so that the denominator will be a perfect square. This can usually be done by inspection, but they should know the rule of multiplying the numerator and demoninator by the same number, usually the denominator. For finding the square root of the numerator the pupil should be allowed to use the table. In the majority of cases these numbers will be less than one hundred.

3. *Radicals.* a. *Teaching the definition of radicals.* A radical may be defined as the indicated root of any number. The number may, however, be rational or irrational. If the number is rational, it may be either positive or negative. Only the indicated roots of positive rational numbers are studied in this unit. The indicated root of a positive rational number may be a rational number or an irrational number. If the indicated root is to be rational, then the number under the radical sign must be a perfect power, as, for example, 9, 16, or $\frac{1}{4}$ if the index of the root is 2, and 8 or 64/125 if the index of the root is 3, and the like. Radicals of this type will give the pupils little trouble if they are explained carefully.

It is the case in which the indicated root of a positive rational number is irrational that will demand most of the effort of teacher and pupils. This type of radical is called a surd. It may have an integer or a fraction under the radical sign.

b. *Teaching the simplification of radicals.* 1′ *When there is an integral expression under the radical sign,* it may be simplified if by factoring one is able to find one or more factors which are perfect squares. When the radicand is an algebraic expression involving literal numbers, reduction of the expression by this method leads to a real simplification. For example $2x^2y\sqrt{xy}$ is simpler than $\sqrt{4x^5y^3}$. However, when the radicand is an expression not involving literal numbers, that is, simply an arithmetical number, it is questionable whether this reduction will give a simpler expression. For example, $\sqrt{75}$ is about as simple as $3\sqrt{5}$, especially if the person who is doing the computing has access to a square root table.

When the number under the radical does not contain any perfect square as a factor, it is often necessary to find the approximate indicated root. This can be done by the method of this unit for square root, but tables must be used for cube root and the like. Later in algebra any required root of an arithmetical number may be found by the use of logarithms.

Practice in evaluating radicals with arithmetical whole numbers would seem to be of slight importance. The skill will become less important as the use of tables becomes more general.

2′ *When there is a fraction under the radical sign,* the process outlined in 2, *i* above will apply. Computation is much easier using this method than any other, and the pupil should be made familiar with it. It is the only way to get the square root of a fractional number except by reducing it to a decimal or by getting the square root of each term and then dividing. The numbers will be small and the tables in the text (39:380) may be used to advantage. It will also serve as an introduction to the more complicated algebraic expressions which may come later.

c. *Teaching the addition and subtraction of radicals.* The two types of simplification mentioned above are useful in the problems on addition and subtraction of radicals. The problems are fairly simple but enable the pupils to see the use of the simplification discussed above. The teacher should try to make

the pupils see that adding like radicals is the same as adding like terms in a or x or y, etc. This will probably not be possible for all the pupils, and too much stress should not be put on it.

Inspection Test No. XXVI, Manual, Page 60, (Course of Study, Tulsa Public Schools, Tulsa, Oklahoma is supposed to be given here and will serve as a part of the testing program for the unit. It covers almost all of the types of problems studied. After the test has been marked, note carefully the errors which the pupils have made and be sure that they understand all or most of these before they are allowed to go on. This reteaching work should be done carefully, and much of it will have to be done with individual pupils. If any large per cent of the class show a lack of knowledge or skill on any particular item, it may be taught again to the whole group. Extra problems are provided in Section V for the better group. These will give these pupils a wider view of the subject while they are waiting for the remainder of the class.

4. *Exponents.* a. *Teaching the meaning of and the relation between exponent and index.* The purpose of this part of the unit is to show the meaning of index and to get the pupil to see how it differs from the exponent idea which he has already. He has been used to seeing the radical sign with no index number written in and it will be necessary to call his attention to the fact that in square root the 2 is understood. From this he can pass to the cube and fourth root of simple numbers and of algebraic expressions. This gives him the meaning of the index number, used largely in the next section on fractional exponents.

b. *Teaching the meaning of fractional exponents.* At this point the pupil is merely told what fractional exponent means. In higher algebra it is defined in such a way that the laws of exponents already laid down will hold. Here one merely tells the pupil what it means and also that the same laws hold as those which were made for integral exponents. This is rigorous enough for the main part of the ninth grade. More careful work may be done with the better part of the group if the time is available. Though it may not be desirable to attempt to show the pupils why we make these particular definitions, the

teachers should have the reason in mind. The three laws or theorems of exponents* are derived directly from the definition of a positive integral exponent. There is no reason to assume that the laws will hold for other sorts of exponents. It is desirable, however, to define other exponents in such a way that the same laws will apply. (57:265) If we assume that Law I ($a^m a^n = a^{m+n}$) holds for fractional exponents, then

$$a^{1/2} a^{1/2} = a^{1/2+1/2} \text{ or } a^1$$

Hence $a^{1/2}$, which is one of the two equal factors of a^1, would be identical with $\sqrt{a}$. Therefore if $a^{1/2}$ is defined as $\sqrt{a}$, Law 1 will hold for this exponent. The general definition of $a^{m/n} = \sqrt[n]{a^m}$, may be made in the same manner. Since only one of the laws of exponents has been used for the purpose of guiding us in making this definition, one is not absolutely sure that all of the laws of exponents may be applied when dealing with fractional exponents. "It is possible, however, to verify by actual test that the powers with fractional, negative and vanishing exponents defined in this way actually obey all the index laws." (57:268) An adequate discussion of this topic is seldom found in college algebras, but such discussions are found in Wilczynski and Slaught's *College Algebra* (57), quoted above and in Palmer and Miser's *College Algebra* (26). Appropriate definitions for negative and zero exponents may be inferred in a similar manner.

c. *Teaching the use of fractional exponents and radicals.* The course of study (Tulsa City Schools, Oklahoma) includes the operations of multiplication and division only with fractional exponents and radicals. To be able to handle these problems it will be necessary for the pupil to be able: 1. to change from a fractional exponent to a radical; 2. to evaluate a numerical example expressed with fractional exponents; 3. to change an expression with a radical sign to an equivalent expression with a fractional exponent; and 4. to apply the laws of multiplication when the exponents are fractions. Each of these skills must be considered by the teacher before he can expect the pupils to succeed with these problems.

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* The laws are expressed symbolically thus: 1. $a^m a^n = a^{m+n}$, 2. $a^m \div a^n = a^{m-n}$, 3. $(a^m)^n = a^{mn}$.

d. *Teaching the meaning of zero and negative exponents.* The meaning of zero and negative exponents should be given to the pupils merely as definitions, as was suggested above in the case of fractional exponents. The exercises included are simple, and no attempt should be made in the first course in algebra to teach the pupils to solve complicated problems. Other like problems may be given for those who need more drill, and more complicated problems for those who will be able to go further in the time available.

Inspection Test No. XXVII (p. 61 in Manual, Course of Study, Tulsa City Schools, 1930, Tulsa, Oklahoma) should be given as a final test. This test deals largely with fractional, zero, and negative exponents.

C. The Use of Supplementary drill exercises and problems. (For a discussion of C see Teaching Unit No. I.)

Section VII. CORRELATIONS

A. With other subjects.
   1. *Shop courses* have some use for radicals and square root.
   2. *In science courses* standard numbers may be used to advantage in many places. This is merely an application of the laws of exponents.

B. Within the subject.
   1. *With equations.* The value of equations is illustrated several places in the unit. They are used first in getting graphs of squares and square roots and also in the application of the Pythagorean relation.
   2. *With geometry.* The study of the theorem of Pythagoras gives an excellent correlation with the informal geometry studied in the earlier grades of the junior high school.
   3. *The practice in the use of tables* of squares and square root will correlate well in many parts of mathematics in which it is necessary to find squares and square roots.

C. Within the unit.
   1. The relation between square root and radicals.
   2. The expression of a fractional exponent as a radical.
   3. Evaluating fractional exponent expressions by the use of square root and other roots.
   4. Evaluating fractional exponent expressions by the use of tables of squares and square roots.

Section VIII. BIBLIOGRAPHY

A. For teachers.

Blackhurst, J. H. Principles and Methods of Teaching Junior High School Mathematics. The Century Company, 1928. pp. 203-209.

This book gives a good discussion of the merits of the two methods of finding square root. The author believes the method of "division" is best.

Englehardt, F. and Heartter, L. D. First Course in Algebra. John C. Winston Company, 1927.

This book treats square root more fully than the textbook (39), and for that reason may be used for supplementary work, both for the good and poor pupils.

Newell, M. J. and Harper, G. A. A Year of Algebra. Row, Peterson and Company, 1920. Chapter VIII.

The work given in this book on roots and radicals is much more complicated than that in the text. Many of the problems will be suitable for the better group.

Palmer, C. I. and Miser, W. L. College Algebra. McGraw-Hill Company, 1928. Chapter VIII.

This book has a brief discussion of roots and radicals and exponents, but it is fully adequate for the teacher's needs.

Smith, D. E. and Reeve, W. D. The Teaching of Junior High School Mathematics. Ginn and Company, 1927.

The chapter on the teaching of algebra gives a good discussion of the purpose of algebra as well as many other suggestions on the teaching of the subject.

Stone, J. C. First Year Algebra. Benj. H. Sanborn and Company, 1928. Chapter VII.

This chapter goes further into the subject of roots and radicals than most textbooks. The problems may be used as supplementary exercises for the better pupils. It also gives a considerable amount of space to the quadratic equation as one of the applications of radicals.

Wells, W. and Hart, W. W. Modern High School Algebra. D. C. Heath and Company, 1923. Chapters XIX and XX.

These two chapters make up quite a thorough discussion of exponents and radicals for the secondary pupil. Much of it can be used as supplementary material.

Wilczynski, E. J. and Slaught, H. E. College Algebra, With Applications. Allyn and Bacon, 1916. Chapter VIII.

This chapter explains fully and rigorously the fractional, zero, and negative exponents as well as irrational and complex exponents. It also shows the relation to the general power function, the exponential function, and logarithms.

B. For pupils.

Englehardt and Heartter. Op. cit. Chapter XIII.
Newell and Harper. Op. cit. Chapter VIII.
Stone, J. C. Op. cit. Chapter VII.
Wells and Hart. Op. cit. Chapter XIII.

Section IX. Testing Program

The testing program for this unit consists of two tests from the Tulsa course of study. (54:60-61) The tests cover the work well and will have to be given anyway. They are to be given at the regular time required by the course of study. The first test may be used as a teaching device as explained above.

Test I.

Find the square root of the following, to one decimal, by division.

1. 170 ( ) 2. 20.8 ( ) 3. 150.4 ( )

Solve the following equations.

4. $4n^2+2=18$ ( ) 6. $\sqrt{x}=5$ ( )
5. $\frac{1}{4}x^2=25$ ( ) 7. $\sqrt{3x+1}=4$ ( )
8. The area of a right triangle is 36 square inches. The base is double the height. What is the hypotenuse? ( )
(make the equation and solve)

Simplify the following radicals.

9. $\sqrt{800}$ ( ) 14. $\sqrt{3/7}$ ( )
10. $\sqrt{261}$ ( ) 15. $\sqrt{9/4}$ ( )
11. $\sqrt{20}$ ( ) 16. $\sqrt{\frac{1}{2}}$ ( )
12. $\sqrt{405}$ ( ) 17. $\sqrt{12}+\sqrt{27}=$ ( )
13. $\sqrt{2/3}$ ( ) 18. $\sqrt{3/4}+\sqrt{4/3}=$ ( )
19. $5\sqrt{12}+\sqrt{75}=$ ( )
20. $\sqrt{24}-3\sqrt{6}+4\sqrt{54}=$ ( )

Test II.

A test on positive, negative, fractional, and zero exponents.

1. $5^0=$ ( ) 4. $-2x^{-1}=$ ( )
2. $3^{-3}=$ ( ) 5. $a^2.a^0.a^3=$ ( )
3. $(-\frac{1}{3})^2=$ ( ) 6. $-X^{5/4}=$ ( )

7. $3A^{2}B^{-1}X^{-2}Y =$ ( )
8. $X^{2} \cdot X^{\frac{1}{4}} =$ ( )
9. $16A^{6} + 2A^{4} =$ ( )
10. $\frac{A^{2}}{A^{-2}} =$ ( )
11. $\frac{8X^{\frac{1}{2}}}{2X^{\frac{1}{3}}} =$ ( )
12. $\frac{8A^{6}}{4^{0}A^{0}} =$ ( )
13. $X^{0} =$ ( )
14. $9^{0} \cdot 4^{\frac{1}{2}} =$ ( )
15. $A^{-1} =$ ( )

TEACHING PLAN NO. V.

Section I. IDENTIFICATION

Teacher: _____ Grade: 7; Date: _____

School: _____ Subject: *Mathematics*

Title: *How to Measure and Draw Angles.* Duration: _____

Section II. SUBJECT MATTER OUTLINE

How to Measure and Draw Angles

A. Meaning and use of Angles
   1. Introduction of the unit
      a. map making
      b. hidden treasure problems
   2. Meaning of angles
   3. Meaning of the size of angles
   4. Symbols for designing angles
   5. The different kinds of angles
   6. How angles are used
      a. in making simple maps
      b. in making indirect measurements
      c. in astronomical measurements

B. Measuring Angles with a Protractor
   1. Units of angle measurement
   2. The protractor and its use
   3. History of angle measurement
   4. Theorems to be discovered by the pupils
      a. vertical angles
      b. base angles of an isosceles triangle
      c. sum of two supplementary angles
      d. sum of the angles of a triangle

C. Drawing Angles with a Protractor
   1. Drawing an angle with a given number of degrees
   2. Drawing an angle equal to a given angle
   3. Bisecting angles
   4. Drawing a perpendicular

5. Using the ability to draw angles
   a. making exaet eopies of triangles
   b. drawing altitudes of triangles
   e. drawing bisectors of the angles of a triangle

Outline for Supplementary Work

A. The Meaning and use of Angles
   1. Hidden treasure problems
      a. using angles
      b. using the loeus idea
   2. Other kinds of angles
      a. complementary angles and supplementary angles
      b. adjacent angles
      c. central angles of a circle
   3. Further uses of angles
      a. to determine the number of degrees in the sum of the angles of any polygon
      b. to determine the relation between the number of degrees in the exter or angle and the two opposite interior angles of any triangle
      c. to draw a regular polygon using a protractor
      d. to make designs
      e. to tell directions
      f. to make circle graphs

B. Construction of Figures with Ruler and Compasses
   1. An angle equal to a given angle
   2. Triangles having given each of the congruence conditions
   3. The bisector of a given angle
   4. A right triangle
   5. An isosceles right triangle
   6. An equilateral triangle
   7. A 30-60 degree right triangle
   8. Designs involving the use of triangles

C. Simple Demonstrations
   1. Prove that the sum of the acute angles of a right triangle is equal to 90 degrees
      a. corollary: If one aeute angle is 60 degrees, the other will be 30 degrees
   2. Prove that the number of degrees in the angles of any polygon is equal to $(n-2)180°$

Seetion III. Pupil Inventory

(See Seetion III Teaehing Plan No. I.)

Seetion IV. Aims or Objectives

A. General Aims
1. To make the pupils familiar with some of the terms used in geometry and in life
2. To give the children an appreciation of the value and use of certain fundamental geometric concepts
3. To develop an appreciation of the beauty of geometric form
4. To develop to some extent the idea of relationship between parts of geometric figures

B. Speeifie Aims
1. Items of knowledge
a. geometric coneepts
(1) angle
(2) size of angle
(3) angles made by intersecting lines and by rotation
(4) symbols for designing angles
(5) kinds of angles, acute, right, and obtuse
(6) units of angle measurement
(7) equal triangles
(8) equal angles
(9) perpendicular to a line
(10 altitudes of a triangle
(11) bisector of an angle
(12) bisectors of the angles of a triangle
b. the meaning and spelling of the following words and significant phrases:

angle	obtuse angle	quarter of a turn
rotation	straight angle	minutes
degree	triangle	protractor
right angle	perpendicular	equal triangles
aeute angle	sides of an angle	seconds
	vertex of an angle	

2. Skills
To develop the following abilities
a. to name an angle in any one of the three ways of making it
b. to measure any angle with a protractor
c. to measure the angles of a triangle
d. to draw an angle with a given number of degrees, using a protractor
e. to draw an angle equal to a given angle, using a protractor
f. to draw a triangle equal to a given triangle

g. to bisect a given angle with a protraetor
h. to draw a line perpendicular to a given line

3. Attitudes or appreciations
   a. an appreciation of the value of angles or direction in loeating places
   b. some appreeiation of the use of angles in field measurements
   c. some appreciation of the use of angles in map making
   d. some appreciation of the value of the use of angles in making many sorts of measurements, both direct and indireet

Section V. Suggested Activities and Exercises

A. Activities

1. *Buried treasure.* This may be made a preliminary discussion to be used to introduce the unit. Show the ways of locating the treasure which depend on the size of the angle, or direction as it is usually ealled. The idea may be carried much further with the better pupils and be made more complicated. Poe's "Gold Bug" and other stories may be used to advantage with this group.
2. *Field protractor.* The children may easily make a protractor which may be used in measuring angles both horizontal and vertical. Descriptions and illustrations of these may be found in many of the junior high school textbooks.*
3. *Simple map making.* Making a map of the school grounds or some other plot is described in one of the references (41:195-223 and will be diseussed more fully later in this plan.
4. *Use of the transit or farm level and sextant.* Angles out of doors may be measured more aecurately if some such instruments are available and the pupils will learn something about how they are used in life. (41:195-223) (42)

B. Exercises

1. For use in regular class work
   (Exereises for regular elass work are always ineluded in the regular teaching unit and may be assumed here.)

* For a general discussion of homemade instruments, see Smith and Reeve, Junior High School Mathematics. (44:359-367)

2. For use as supplementary exercises to care for individual differences
   a. for remedial purposes
      (These problems should be nearly like the ones used in the unit and may also be assumed.)
   b. for the better group of pupils
      (The outline for this material is included in Section II above. The books referred to below are standard secondary school textbooks. The range is purposely wider than any one pupil will be able to cover in order to allow for selection.)

Supplementary work for the better group

A. The meaning and use of angles
   1. Hidden treasure problems
      a. using angles
         Stone, Book I*, p. 117. Schorling and Clark, Seventh Year, p. 71, problems 1-7.
      b. using the locus idea
         Smith and Reeve, The Teaching of Junior High School Mathematics, pp. 318-320.
   2. Other kinds of angles
      a. Complementary and supplementary angles
         Betz, pp. 38-39, problems 1-14.
      b. adjacent angles
         Betz, p. 26. Breslich, Book I, pp. 103-104, problems 1-8.
      c. central angles of a circle, arcs, and chords
         Betz, pp. 35-36.
         Breslich, Book I, pp. 260-262, problems 1-4.
         Gonnelly and Huff, Seventh Year, pp. 77-78, problems 1-6.
   3. Further uses of angles
      a. to determine the number of degrees in angles of any polygon. (give as a demonstration)
      b. to determine the relation between the number of degrees in the exterior angle of a triangle and the two opposite interior angles.
         Betz, p. 45, problems 1-12.
      c. draw a regular polygon using a protractor
         Strayer-Upton, Book I, p. 205.
         Schorling and Clark, Seventh Year, pp. 123-125, problems 1-8.
         Breslich, Book I, p. 108, problems 1-8.

* For full titles of these books see Section VIII.

d. to make designs
Strayer-Upton, Book I, p. 206, problems 1-6.
Breslich, Book I, p. 263.
e. to tell direction
Strayer-Upton, Book I, p. 209, problems 1-6.
Betz, pp. 31-32.
Schorling and Clark, Seventh Year, pp. 72-73.
Gonnelly and Huff, Seventh Year, pp. 93-95.
f. to make circle graphs
Strayer-Upton, Book I, p. 214, problems 1-3, p. 216, problems 1-10.
Schorling and Clark, Seventh Year, pp. 126-130, all the problems.

B. Construction of figures with ruler and compasses
1. An angle equal to a given angle
Stone, Book I, p. 125, problems 1-2.
Breslich, Book I, p. 195.
2. Triangles having given each of the congruence conditions
Schorling and Clark, Eighth Year, pp. 62-65.
Stone, Book II, pp. 73-75.
Strayer-Upton, Book ii, pp. 223-226.
3. The bisector of a given angle
Schorling and Clark, Seventh Year, p. 124.
Stone, Book II, p. 62.
Strayer-Upton, Book I, p. 201.
4. A right triangle
Schorling and Clark, Seventh Year, p. 121.
Strayer-Upton, Book II, pp. 214-215.
Wentworth-Smith-Brown, Book I, pp. 152-156.
5. An isosceles right triangle
Stone, Book I, p. 131, problems 1-4.
Betz, pp. 46-48, problems 1-15.
6. An equilateral triangle
Strayer-Upton, Book I, p. 197.
7. A 30-60 degree right triangle
Stone, Book I, p. 127.
8. Designs involving the use of triangles
Strayer-Upton, Book I, pp. 206-207, Book II, p. 218.

C. Simple demonstrations

Work of this sort will be found in only a few of the junior high school mathematics textbooks at present. (48:221) (46:64-69) (34:39-43) It is possible, however, to get pupils of the junior high school age to think logically if enough

assumptions are made. In the first case mentioned in the Outline for Supplementary Work (C, 1) Section II, if the pupils know that the sum of the angles of any triangle is 180° and that one angle is 90°, they will see that the sum of the other two angles is 90°. This will be deductive thinking though it is simple. This type of work can be done easily by pupils of the better part of the class. The teacher will have to decide just how much of this demonstration will be possible and desirable. It seems that this way of introducing demonstrative geometry is better in many ways than the plan which introduces it all in one block. It is at least worth trying and this unit is suggested as one of the places in which simple demonstrations may be introdueed.

Section VI. Teacher Procedure and Means

Outline

A. The use of the teaching plan during teaching
B. Detailed procedure in teaching the unit
C. The use of the supplementary exercises and problems

A. The use of the teaching plan during teaching

(For a discussion of A see Teaching Plan No. I.)

B. Detailed procedure in teaching the unit.

1. *Introducing the unit.* The introduction of this unit should give the pupil some idea of the need for angles, how they are used, and something about how they are drawn and measured. In other words the pupil will be given in the beginning a general overview of the topic which will tend to orient him with respect to it. Some suggestions are given as to how this may be done. The work will necessarily be superficial and should be presented more carefully later during the unit. Probably one of the best ways to do this is to make with the children a crude map of the school ground by the use of a plane table and alidade; this is fully described elsewhere by Shuster, (40:215-217) and will not be discussed here. This also correlates closely with the work the pupil does in map making in social science or geography. In this work pupils see one important use of angles; and if the map is transferred

from the original paper, they can see some reasons for making or wanting to make angle measurements and for drawing angles of various sizes.

The unit may also be introduced by a consideration of hidden treasure problems, not at all a practical type of problem but a type very fascinating to pupils in the seventh grade. This could well be elaborated into a teaching unit in itself with the chief emphasis on the locus idea. This phase will be played up to some extent in the supplementary work for the better pupils of the class. Here all one is interested in is that they see that the location of the spot where the treasure is buried depends upon both distance and direction, and that the direction is an angle measured from some line of reference. They may then be made to see that there must be some way to measure angles on the actual spot and to construct angles of the given size on the chart or map of the location to be used as a guide in finding the treasure.

2. *Meaning and use of angles.* a. *The idea of an angle.* It is useless to attempt to define an angle for any student in the secondary school. (45:251) Either of the activities suggested above, if carried out carefully, will give the pupils some idea of the meaning of an angle. At first they may be merely told that an angle is formed when two lines meet at a point or that it is the corner of familiar plane figures. They should be asked to pick out angles in the school room and outside.

b. *The idea of the size of an angle.* For the pupil to understand the meaning of size of angles the idea of an angle as *rotation* must be presented. Probably the best way to do this is to use the rotation of the hour hand of a clock or the indicator arm above the average elevator or some such familiar illustration. Another good example to use is the farm level or transit if one is available. The rotating of the alidade used in making the map referred to above is another illustration.

c. *Symbols for designating angles.* The three ways of naming an angle are important and very often confusing to children. It is important then that these be taught and practiced carefully at the beginning of the junior high school course.

Tests included in Section IX will tell whether the pupils have acquired this skill.

d. *The kinds of angles.* Acute, right, and obtuse angles are the ones of most importance. The right angle should probably be defined as a square corner; the pupils might also be asked to note the amount of rotation it represents. The concept of a straight angle will be more real to the pupil if discussed in connection with the sum of the angles of a triangle. For all of the pupils there seems to be little justification for discussing the other kinds of angles here. However, discussion and problems concerning them will serve as excellent material for supplementary work.

e. *How angles are used.* Many uses of angles may be discussed with the pupils. The ones chosen should vary with the local conditions, the textbook, and reference books available to the pupil. Here shall be discussed three uses as suggestions, the use in simple map making, the use in simple indirect measurement, and the use in making astronomical observations. It would probably be advisable to make a better map perhaps by the method of triangulation, than the one suggested in the beginning. In transferring the map to another paper most of the things already learned in the unit will be of use. Measuring the height of trees, buildings, and the like and the distance across lakes or rivers is a practical use of angles in which the pupils will be interested. This should be treated only briefly since it will be taken up more fully later on. The pupils may be told that the surveyor can measure any side of a triangle with the use of another side and the angles. They may be told of the isosceles right triangle method and given some practice in its use. Members of the class will be able to make such right triangles and can do measuring independently.

The use of angles in astronomical calculation involves the use of a considerable amount of trigonometry as well as other mathematics. It will, therefore, be impossible to do more than tell the pupils that distance to the planets and other heavenly bodies may be determined by measuring certain angles and lines. Most children of this age will be interested in talking about these measurements and will be able to see the essential

advantages of this use of angles. This will be especially true if some reeent discovery has stirred the popular mind, as, for example, the diseovery of the trans-Neptune planet.

3. *Teaching the measurement of angles.* a. *Teaching the units of angle measurement.* The pupils have learned the meaning of 'right angle'. The degree may be defined as one ninetieth of a right angle. The pupils may then be shown that a complete eircle or rotation would be 360°. At this time it would probably be advisable to say something about the origin of the degree as a unit of measurement.

b. *The protractor and its use.* There should be a protractor for each member of the elass. The elass may be told that the protraetor is an instrument to be used in measuring the size of angles in a manner similar to the way the ruler is used in measuring the length of lines. The use of the protraetor in measuring angles will tend to clarify and extend other coneepts previously taught. The idea of the degree as the unit of angle measurement will stand out more elearly in the mind of the pupil after he has measured many angles of different sizes. The coneept of the size of an angle will be mueh extended by measuring angles of various sizes.

The preliminary work in the use of the protractor might well be made up of exereises in which the pupils determine whether the given angle is acute, right, or obtuse. Later have the pupils measure angles more accurately. Numerous exercises should then be given in the measurement of angles of all sorts and in estimating the size of given angles and cheeking by measurement. These measurements should be made on angles in various positions. It should also be pointed out that in much of the practical work tenths and hundredths of degrees are used instead of minutes and seconds.

e. *The history of angle measurement.* Something may be said about the history of angle measurement as a matter of interest and give the pupils some idea of how the units of measurement have developed. They will be interested to see the influence of the Babylonian astronomers on present day units of measures.

d. *Theorems or facts discovered by angle measurement.* A number of theorems or facts may be diseovered by the pupils with the information and skills they now have. The amount of work done on this phase of the unit will depend upon the time the teaeher has at his disposal and upon the ability of the elass. A few theorems will be suggested and discussed here. Others may be added by the teaeher if it is found desirable. The truth of the two theorems about opposite or vertieal angles and the base angles of an isoseeles triangle may be readily admitted by the pupils but may be eheeked by measurement merely as an exereise in the use of the protraetor. Two adjaeent angles whose exterior sides form a straight line may be measured to give a preliminary meaning of a straight angle. The ehildren shonld be told that though an angle of 180° may not seem like an angle at all it is really a most important angle. This will lead up to the most important theorem in this group, the sum of the three angles of a triangle. The following procedure is suggested; have eaeh of the pupils draw a triangle of any kind on his paper and then let eaeh measure the three angles of his triangle and find the number of degrees in the sum. These sums may be plaeed on the board in the form of a table. If the sum obtained by any pupil is mueh above or below 180°, have him eheck his measurements before the result is plaeed in the table. Pupils will easily arrive at the eonelusion that the sum of the angles of any triangle equals 180°. The proof by tearing off the eorners of the triangle and fitting them together to make a straight line is easily performed and usually eonvincing.

4. *Teaching the drawing of angles with a protractor.*

a. *Drawing an angle having a given number of degrees.* The drawing of angles and triangles should begin with a working line drawn on the board or paper. Mark some beginning point on this line and work from that. To draw an angle with a given number of degrees the pupil will have to learn to place the zero point of his protraetor on this point on the working line, then read the given number of degrees on the proper seale of the protraetor, mark the point, and with a ruler eonneet it with the point oeeupied by the zero point of the protractor.

The pupil should see that this makes an angle of the proper size.

b. *Drawing an angle equal to a given angle.* Begin with a working line as before. The only new thing here will be to know to measure the angle, and then the remainder of the problem will be the same as in a above.

c. *Bisecting an angle.* Teach the meaning of bisect and show the pupils that the half angle is found by dividing the size of the whole angle by 2; the work is the same as a above.

d. *Drawing a perpendicular.* In like manner after the pupils see what a perpendicular is they merely have to draw an angle of 90° at the required point.

e. *Use of angle drawing.* In transferring a map made by triangulation to another paper it will be neeessary to make triangles equal to the ones on the original map. It may be assumed that if the base and the two adjaeent angles are made equal the triangles will be just alike. Some praetiee should be given until the pupil is quite familiar with the method of making two triangles equal. In the same way practiee should be given in making triangles equal by having two sides and the included angle equal. Drawing the altitudes of a triangle and the bisectors of the angles of a triangle may be given as interesting exercises. The pupils should be able to discover for themselves that the lines are coneurrent.

C. The use of supplementary drill exercises and problems. (For a diseussion of C see Teaching Plan No. I).

Section VII. Correlations

A. With other subjeets.

1. *With social science or geography.* The making of simple maps as discussed above in Seetion VI will add much to the study of maps in geography or social scienee. The use of angles in reading directions and the connection between degrees of latitude and Standard Time are less valuable but would be interesting to some of the pupils.
2. *With manual and industrial arts.* Knowledge of different kinds of angles, how they are drawn, measured,

bisected, and the like would be of value in many types of woodwork and in much of the industrial work. For example, in making a miter-joint in woodwork the pupil will have to know the angle needed and how to set the miter box to that angle or how to draw the angle on the wood. This is usually a right angle or a 45° angle, but it may also be of other sizes. Dove-tail joints require the measurement and construction of angles.

In cutting a board to an angle with a circle saw it is necessary to set the guide on the machine at the proper angle. The same result may be accomplished by drawing an angle on the board and sawing along the line; this would not be done with a circle saw, however.

B. Within the subject.

1. *With later work in informal geometry.* The work on angles is the ground work for much of the later work in informal geometry. This is seen to be true by glancing at the list of topics in geometry usually taught in the junior high school course; measurement, graphs, constructions, areas, volumes, and right angles.
2. *With work on numerical trigonometry and indirect measurement.* For example, the tangent of an angle is the ratio of the opposite side to the adjacent side of a right triangle containing this angle. If the angle changes, the value of the ratio changes. Therefore, the pupil must have at the beginning a clear concept of an angle and its size. From this it is possible to build the concept of the function of an angle and other phases of numerical trigonometry and indirect measurement.
3. *With demonstrative geometry.* The fundamental ideas of angles and of angle measurement and construction learned here will be applicable in plane geometry later. The same is true of most of the concepts and terms used in this unit. Before a pupil can get anything out of a construction problems calling for the construction of one angle equal to a given angle, he must have a concept of an angle and its size. The idea of an angle and its size is also essential when the pupil demonstrates many of the theorems of plane geometry, as, for example, proving alternate interior angles equal, the sum of the angles of a triangle equal to two right angles, and the like.

C. Within the unit.

1. The discussion of the meaning and use of angles should make the pupils interested in the measurement of angles and in the instrument for making measurements, the protractor.
2. In like manner the measurements of angles should be made to correlate directly with the construction of angles with a protractor.

Section VIII BIBLIOGRAPHY

A. For teachers.

Hassler, J. O. and Smith, R. R. The Teaching of Secondary Mathematics. The Macmillan Company, 1930.

This book is good for a general discussion of the teaching of secondary mathematics. Part I deals with the primary concepts of mathematics, and Part II with teaching problems in various parts of secondary mathematics.

National Council of Teachers of Mathematics, Third Yearbook. Teachers College Bureau of Publications, 1928.

This book contains many good articles on the teaching of secondary mathematics. A few of them have been referred to above. They are all valuable to the teacher of mathematics, and some of them will be especially helpful to the work of this unit.

_____Fifth Yearbook. Bureau of Publications, Teachers College, 1930.

This yearbook is devoted entirely to the various phases of the teaching of geometry, and it contains a few excellent articles on the part that geometry should play in the work of the junior high school. One of these has been referred to above.

Schultze, Arthur. The Teaching of Mathematics in Secondary Schools. The Macmillan Company, 1912.

This book is written from the standpoint of the senior high school, but it furnishes some excellent material on how to teach the meaning of angles.

Smith, D. E. and Reeve, W. D. The Teaching of Junior High School Matematics. Ginn and Company, 1927.

This is probably one of the best all around books available on the teaching of junior high school mathematics. Chapter V is on the teaching of intuitive geometry, and Chapter XII is on the construction and use of simple instruments for measuring angles and the like.

Textbooks.

The teacher can get much help in the material to be taught and in its organization and in the methods of teaching various topics from a study of various textbooks in junior high school mathematics. A number of these are listed under B below.

B. For pupils.

Betz, Wm. Geometry for Junior High Schools. Rochester, New York: Board of Education, 1928.

Breslich, E. R. Seventh Year Mathematics. The Macmillan Company, 1929. Chapter III.

Gonnelly and Huff. Seventh Year Mathematics. Charles E. Merrell Company, 1925.

Poe, Edgar Allen. The Gold Bug.

Schorling, R. and Clark, J. R. Modern Mathematics, Seventh School Year. World Book Company, 1929. Chapter III.

Stone, J. C. The New Mathematics, Book I. Benj. H. Sanborn and Company, 1926. Chapter V.

Strayer, G. D. and Upton, C. B. Junior Mathematics, Book I. American Book Company, 1929. Chapter XII.

Section IX. Testing Program

As a testing program for this unit six tests from a standard test and exercise book (45) are recommended. The tests are 1. Test No. 8 "Familiar Types of Angles", 2. Test No. 81 "Measuring Angles", 3. Test No. 83 "Measuring Angles", 4. Test No. 82 "Drawing Figures With Ruler and Protractor", 5. Test No. 89 "Bisecting Angles", and 6. Test No. 90 "Constructing an Angle Equal to Another". More tests are included than will ordinarily be given to one class. This is done to show how such test books may be used in connection with a teaching plan. The teacher may select the ones which he wishes to use for any particular group. The tests may not cover the items of the unit exactly, but they are fairly complete and will answer the purpose very well. When such tests as these are available, they save much work on the part of the person who is planning the unit.

TEACHING PLAN NO. VI.

Section I. IDENTIFICATION

Teacher _____ Grade: *9-A* Date: _____
School: _____ Subject: *Mathematics*
Title: *Demonstrative Geometry for Ninth Grade*
Duration: *8 weeks.*

Section II. SUBJECT MATTER OUTLINE
DEMONSTRATIVE GEOMETRY FOR THE NINTH GRADE

Syllabus

Chapter I. Fundamental Ideas and Elementary Constructions

1. Meaning of geometry
2. Brief history of geometry
3. Terms and definitions, (point, straight line, circle, center, radius, arc, and diameter).
4. Problem A. On a given line mark off a line equal to another given line.
5. Problem B. Bisect a given straight line.
6. Definitions, (triangles, sides, vertices, base, and vertex).
7. The meaning of an angle.
8. Problem C. At a given point on a line construct an angle equal to a given angle.
9. Problem D. Bisect a given angle.
10. Definitions, (isosceles triangle, arms, base, vertical angle, adjacent angles, perpendicular, and right angle).
11. Problem E. At a point on a line construct a perpendicular to the line.
12. Problem F. From a point outside of a line construct a perpendicular to the line.
13. Miscellaneous construction exercises.

Chapter II. Development of Demonstration

1. Discussion and motivation of demonstration.
2. Definitions, (axiom, postulate, theorem, problem, and proposition).
3. Informal proofs of simple exercises.
4. Definitions, (right angle, obtuse angle, acute angle, degrees, minutes, seconds, supplementary angles, and adjacent angles).
5. Theorem A. When one straight line meets another the adjacent angles thus formed are supplementary. (prove informally)

6. Theorem B. If two angles are supplementary to the same angle, they are equal. If two angles are supplementary to equal angles, they are equal. (prove informally)
7. Exercises dealing with supplementary angles. (involving proof).
8. Formal proof of exercises under 7.
9. Theorem C. When two straight lines intersect, the opposite angles are equal. (the pupil to give a formal proof).
10. Definitions, (complementary angles, complement).
11. Theorem D. If two angles are complementary to the same angle, they are equal. If two angles are complementary to equal angles, they are eqnal. (informal treatment only).
12. Definitions, (angles adjacent to the side of a triangle, and the angle included between two lines).
13. Theorem E. If two triangles have two sides and the included angle of one equal respectively to two sides and the included angle of the other, they are equal. (treated experimentally only).
14. Theorem F. If two triangles have one side and two adjacent angles of one equal respectively to one side and two adjacent angles of the other, they are equal. (treated experimentally only.)
15. Definitions, (equilateral triangle, equangular triangle, right triangle, obtuse triangle, acute triangle).
16. Exercises having the figures drawn and the hypothesis an conclusion stated, to give a formal proof.

Chapter III. Formal Demonstrative Geometry.

1. Miscellaneous exercises using the two congrnence theorems stated and proved experimentally in Chapter II.
2. Theorem I. The opposite angles of an isosceles triangle are equal.
3. Definitions, (parallel lines, corresponding angles, and transversal).
4. Theorem II. If two straight lines are cut by a transversal so that the alternate interior angles are equal, the lines are parallel. (formal proof).
5. Theorem III. When two parallel lines are cut by a transversal, the alternate interior angles are equal. (formal proof).
6. Theorem IV. When two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary. (formal proof).
7. The sum of the angles of a triangle is equal to two right angles. (formal proof).

8. Two triangles are similar if at least two angles of one are equal to two angles of the other. (treated experimentally only.
9. Definition, (corresponding sides).
10. Theorem VII. When two triangles are similar, the corresponding sides are in proportion. (treated experimentally only).
11. Theorem VIII. Corresponding altitudes of similar triangles have the same ratio as any two corresponding sides. (formal proof).
12. Theorem IX. In any triangle the altitude on the hypotenuse divides the triangle into two triangles each similar to the given triangle.
13. Theorem X. In any right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Outline of Supplementary Work

A. For the better group of pupils
   1. Projects
   2. The nature of a proof
   3. The nature of a geometric sequence
   4. The analytic method of proof
   5. Applications of geometry.

Section III. Pupil Inventory

(See Section III, Teaching Plan No. I.)

Section IV. Aims or Objectives

A. General aims.
   1. To show the pupils the "significance of a demonstration".
   2. To give the pupils some idea of the nature of tenth grade geometry.
   3. To show the pupils the relation of informal to demonstrative geometry, including both likenesses and differences.
   4. To bring to consciousness certain methods or ways of reasoning.
   5. To establish the confidence of the "pupil in his own ability to reason from one fact to another and set down his reasoning in a business-like way."

B. Specific aims.
   1. Items of knowledge
      a. The meaning and spelling of the following words and significant phrases

point
straight line
circle
center
radius
arc
diameter
triangle
sides
vertices
base
vertex
isosceles triangle
altitude
vertical
axiom
postulate
theorem
problem
right triangle
obtuse triangle
acute triangle
adjacent angles
degrees
minutes
seconds
supplementary angles
complementary angles
complement
angles adjacent to a side of a triangle
the angle included between two lines
equilateral triangle
equiangular triangle
transversal
alternate interior angles
corresponding angles
similar triangles
proportion
corresponding sides
ratio
hypotenuse

b. The definitions of the following terms and phrases will be needed as authority in demonstrations.
(definitions to be memorized by the pupils)
supplementary angles
complementary angles
right angles
parallel lines

c. Axioms and postulates. (to be memorized by the pupils)
(1) Things which are equal to the same thing are equal to each other. (axiom)
(2) Things which are equal to equal things are equal to each other. (axiom)
(3) All right angles are equal. (axiom)
(4) Corresponding sides and angles of equal triangles are equal. (postulate)
(5) A quantity may be substituted for its equal in any expression without changing the value of the expression. (axiom)
(6) If equal quantities are added to equal quantities, the sums are equal quantities. (axiom)
(7) If equal quantities are subtracted from equal quantities, the remainders are equal quantities. (axiom)

(8) The whole is equal to the sum of all its parts. (axiom)

d. Theorems. (to be memorized by the pupil) (All of the theorems which are proved or postulated should be memorized by the pupils while they are studying the particular theorem. They will be needed as reasons in the other demonstrations which follow. The list of theorems will be found in the syllabus of Section II above).

2. Skills

a. The ability to do the following construction exercises.

(1) On a given line mark off a line equal to a given line.

(2) Bisect a given straight line.

(3) At a given point on a line construct an angle equal to a given angle.

(4) Bisect a given angle.

(5) At a point of a given line construct a perpendicular to the line.

(6) From a point outside a line construct a perpendicular to the line.

b. The ability to show experimentally the truth of the following theorems.

(1) If two triangles have two sides and the included angle of one equal respectively to two sides and the included angle of the other, they are equal.

(2) If two triangles have one side and two adjacent angles of one equal respectively to one side and two adjacent angles of the other, they are equal.

(3) Two triangles are similar if at least two angles of one are equal to two angles of the other.

(4) The corresponding sides of two similar triangles are proportional.

c. The ability to give an informal proof for the following theorems.

(1) When one straight line meets another, the adjacent angles thus formed are supplementary.

(2) If two angles are supplementary to the same angle, they are equal. If two angles are supplementary to equal angles, they are equal.

(3) If two angles are complementary to the same angle, they are equal. If two angles are complementary to equal angles, they are equal.

d. The ability to demonstrate the following theorems, giving a formal proof.

(1) When two straight lines intersect, the opposite angles are equal.

(2) If two straight lines are cut by a transversal so that the alternate interior angles are equal, the lines are parallel.

(3) If a triangle is isosceles, the angles opposite the equal sides are equal.

(4) If two parallel lines are cut by a transversal, the corresponding angles are equal.

(5) The sum of the angles of a triangle is equal to two right angles.

(6) Corresponding altitudes of similar triangles have the same ratio as any two corresponding sides.

(7) In any triangle the altitude upon the hypotenuse divides the triangle into two triangles each similar to the given triangle.

(8) In any right triangle the square on the hypotenuse is equal to the sum of the square on the other two sides.

e. The ability to prove original exercises.
(Many original exercises are given using the theorems proved. Some of these are to be proved informally and the others formally. The pupils should be able to do the majority of them).

3. Attitudes or appreciations.

a. An appreciation of the meaning of logical thinking.

b. An appreciation of the use of logical thinking in geometry.

c. An appreciation of the fact that the same type of thinking is applied in daily life.

d. An appreciation of "the worth of geometry in its vital connection with the development of useful learning in the progress of civilization."*
(13:301)

* From Hassler & Smith, The Teaching of Secondary Mathematics. By permission of The Macmillan Company, publishers.

Section V ACTIVITIES AND EXERCISES

A. Activities

1. *A discussion of symmetry* shows one reason why one needs to make certain constructions that are included in the introductory chapter of this unit. For example, a symmetrical figure has an axis of symmetry, and to draw such an axis one must be able to construct the perpendicular bisector of a line. In other symmetry problems it will be necessary to drop perpendiculars from a point, construct angles equal to other angles, and the like. A thorough discussion of symmetry as applied to elementary geometry would require numerous drawings and explanations and will not be included here. (6:1-19) (59:145-148)
2. *The use of applied problems in geometry.* The use of simple applications of the theorems studied help in seeing the meaning as well as the value of the theorems. Some such exercises are found in the text of the unit.* Others can be found in modern textbooks on plane geometry.
3. *The accurate and careful construction of figures.* Many pupils may be interested in constructing carefully plates containing related figures studied in the unit. This correlates directly with mechanical drawing. Several plates could be selected from the entire course and much could be learned in this way. . Beneath certain figures space could be left and neat model demonstrations included. If these plates are exhibited to the class and the school at large, more pride will be taken in the work.
4. *Geometric posters.* Though this unit takes up a limited number of theorems, it is possible for interested teachers and pupils to find several interesting things to use for poster material. Probably the most outstanding are the Pythagorean Theorem and the Pons Asinorum or "Bridge of Asses." (35:270-272) (16:268-276) The posters should be exhibited outside also.

B. Exercises.

1. *For use in regular class work.*
   (Exercises for the regular class work are included in the text of the unit).
2. For use as supplementary exercises.
   (to care for individual differences)

* The subject matter of this unit was taken largely from a textbook on plane geometry, but the organization is different in some particulars; it will be referred to as the text of the unit.

a. *For remedial purposes.* (No exercises will be included under this head. There are probably more exercises throughout the unit than the average teacher will want to use. A few theorems and exercises properly understood will probably give the meaning of a demonstration, the principal aim of the unit, but many theorems and exercises totally or partially understood will probably have a negative effect).
b. *For the better group of pupils.*

(1) *Projects.* Most of the activities listed above are of the type applicable to the better pupils. A pupil may be working on one or more of them at the same time, as his needs require.
(2) *The nature of a proof.* The better pupils will be able to gain a good appreciation of the nature of a geometric proof. It is desirable, therefore, to stress formal proof more with this group than would be done in the regular class work.

A formal proof must have the hypothesis and conclusion clearly stated, and each statement in the proof must be in logical order and supported by either an axiom, a postulate, a definition, or a previously proved proposition. Moreover, these statements must be of such a nature and arrangement as to lead from the hypothesis to the conclusion. Knowing methods of proof enables one to prove a statement beyond the possibility of a doubt. A person can then convince others of what he knows to be true and test out theories and hypotheses leading to the discovery of new truths. (6:28)

(3). *Nature of a geometric sequence.* The nature of a proof, discussed above, is directly connected with the fundamental assumptions and previously proved theorems. The good pupil will be able to see the fundamental connection between the assumptions and the sequence of theorems much better than the others, and special effort should be made to give him the opportunity.

With this group it would be in place to discuss the undefined terms, the properties of a good definition, and even demonstrations of some of the postulated theorems.

(4). *The analytic method of proof.* The method of analysis cannot be fully described here. "But to be truly initiated into

the spirit and meaning of logical processes, he (the pupil) must acquire mastery that comes only from analytical thinking . . . . . Only when he has received sufficient training at attacking confidently a new proposition or problem, starting with the conclusion, working backward step by step to his previously established body of truth, can he be said to have mastered the spirit of geometric reasoning." (41:134-144)

Because of the brief period which the pupils study demonstration in the ninth grade, the wisdom of introducing the analytic method may be doubted. One could scarcely hope that the whole class would gain a comprehension of it in six or eight weeks. (41:142-143) For the better group of pupils more is possible, and teachers can well consider its use. The selection of actual cases in which the analytic method can be applied to advantage can be left to the individual teacher. If the teacher understands analysis as applied to geometry and appreciates its value as a means of discovery, he will have little trouble in deciding when it can be used advantageously. The numerous exercises that follow the theorems on congruence and the isosceles triangle offer abundant material for the use of the analytic method. (44:134-144) (60:54-56)

The main difficulty here, as in the other supplementary projects mentioned above, is for the teacher to get his work so organized that he will have time to discuss these things with the better group of pupils. It is not sufficient to have the pupils read about them. Carefully guided practice is needed. Much can be done by suggestions made to individual pupils during supervised study periods. However, the teacher should not expect too much from the pupils in regard to these projects in the short time that will be available. They are aims that are often not accomplished after a year spent in the study of geometry.

(5) *Applications of geometry.* Practical problems using the theorems proved in this unit will furnish much supplementary material. These problems involve the ability to translate word statements into geometric statements, which is not one of the primary aims of this unit. It may well be expected of the better group of pupils, however. It is not necessary to

quote any of these problems here, since they can be found in abundance in modern textbooks on plane geometry.

Section VI. Teacher Procedure and Means

Outline

A. The use of the teaching plan during teaching
B. Detailed procedure in teaching the unit
C. The use of the supplementary drill exercises and problems

A. The use of the teaching plan during teaching
(For a discussion of A see Teaching Plan No. I)

B. Detailed procedure in teaching the unit.

The aim of a unit in demonstrative geometry for the ninth grade may well be discussed before taking up the discussion of the detailed teaching procedure. It seems reasonable to suppose that if any demonstration is to be given in the junior high school that much of it can be introduced at opportune places throughout the three grades. Some of the preceding teaching plans contain suggestions showing how and where this may be done.* Dr. Sanford has described demonstrative work done by seventh and eighth grade pupils in the Lincoln School of Teachers College. (34:39-43) The present unit however, is not based on any such previous work.

A unit of demonstrative geometry for the ninth grade has two principal aims, first, "to show the pupil what 'demonstration' means." (30:25) This meaning of a demonstration is taken to imply an appreciation of rigor in mathematics and the real significance of mathematical thought, which includes the methods of synthesis and analysis. (44:231) The second aim is to show the pupils something of the nature, content, and possibilities of later courses in mathematics and "to give each pupil an opportunity to determine his aptitudes and preferences therefor." (30:26) The results of some work in demonstration during the junior high school period also enables the teacher to advise the pupils more intelligently as to their future work in mathematics.

* See Teaching Plan No. I, for grade 7, Section V under Supplementary Drill Exercises and Problems, C. See also Teaching Plan No. III, for grade 8, under Supplementary Drill Exercises and Problems. B, 2, b (2).

Unless the demonstrative geometry taught in the junior high school shows the pupil the meaning of *demonstration* and aids the pupil and teacher in choosing future work, there is little or no justification for including it in the course of study. (2:123-128) For this reason in teaching the unit these aims should be kept uppermost in mind, and both the subject matter presented and the methods of teaching used should be so chosen that these aims may be best attained.

I. Introduction

1. *Aim of the introductory chapter.* The introductory chapter will deal largely with construction problems and exercises. The reasons for doing this are: *a.* To allow the pupil to become partly familiar with the mathematical language necessary for the study of geometry; *b.* to give the pupil ability to construct many of the figures he will use in later demonstrations of the unit; *c.* to help the pupils "see the essential relationships of the parts of a geometric figure," for example, that the side or angle of a triangle may be bisected in any position; *d.* to show, while still dealing with concrete figures, the meaning of "given" and "to prove" and other such concepts; *e.* to show the teacher the smallness of the gap which the ordinary pupil is able to jump in learning new concepts.

If these things can be accomplished by introductory work dealing with construction problems, then certainly these problems are worth while. It is reasonable to suppose that in a junior high school in which a unit of demonstrative geometry is taught, much of this material will have been covered in the two previous grades. In such a case it will probably be advisable to omit all or parts of the first chapter. However, in this chapter more emphasis is laid on correct expression and concise thinking than is the case in the work done in grades seven and eight. It gives a gradual, connected development of accurate language and concepts that a pupil could hardly get in the earlier part of the junior high school course. A test will be included (Test I, Section IX) to aid the teacher in estimating the amount of time to be spent on this chapter. The discussion that follows under 2, 3, 4, 5, 6, and 7 will not be applicable in case the introductory chapter is omitted.

2. *The meaning of geometry* is given by showing its relation to eoneepts and objects with whieh every one is familiar. Work done in informal geometry may be referred to here. In all eases the pupils have had something to do with areas and volumes, if nothing else. Nothing is said of demonstration, the primary purpose of the unit, since the first ehapter is entirely intuitive and is ineluded for the purpose of creating a ground work on which to build the demonstrative work.

3. *Teaching the meaning and method of making constructions.* "Construction" in geometry means construction by means of the straight edge and compasses. This fundamental meaning of construction should be emphasized throughout the unit. This will offer no great difficulty, since ordinarily the pupil will not consider the use of other instruments. The methods of making various constructions are explained and diseussed in the text of the unit and need not be repeated here.

4. *Teaching definitions.* a. *Definitions should be understood* by the pupils. No pupil gets his idea of the meaning of a geometric figure from a formal definition of it. (44:251) For this reason the text of the unit goes to some effort to see that the pupil understands the meaning of each term before any formal definition is offered. Exercises and questions follow the definitions for the purpose of giving the pupil an opportunity to become acquainted with the terms by using them. They are intended not only to "emphasize the meaning of the terms, but to call forth some independent thought about them."* (13:230)

It will be noted also that the definitions in this unit are given immediately preceding their use in problems or theorems. That is, no definition is given until it is needed in some of the work of the unit.

b. *Memorizing definitions.* More attention should be paid to the ability to repeat definitions in geometry than in other parts of mathematics or in other sciences. This is true beeause of the fact that in geometry many definitions are used as reasons for statements made in demonstrations. When this is

* From Hassler & Smith, Op. cit. By permission of The Macmillan Company, publishers.

done, it is desirable that the person giving the proof know an accurate, concise definition of the term or concept in question. As examples of terms whose definitions must be referred to in proving propositions, we have perpendicular, right angle, supplementary angles, and the like. A list of the terms whose definitions should be memorized will be found in Section IV of this plan. Other definitions need not be memorized.

c. *Precision of definition.* Not all terms should be defined precisely. In fact it is usually not possible to do so for the secondary pupil. Concepts concerning many of the simplest geometric figures must be changed or extended as we go further into the study of the subject. A quadrilateral may be defined as "a closed figure formed by four straight lines" (27:92) and because of illustrations given in the text the pupil would rightly interpret it to mean a convex figure. However, it does not mean precisely that, and "the teacher should realize that in general a quadrilateral is formed by four straight lines lying in a plane. If one adopts modern phraseology, he says that they intersect in point at a finite distance or at an indefinite distance, and that in any case we have three diagonals." (44:250) A consideration of this would suggest another reason for not memorizing definitions that are not actually used in proving propositions.

The teacher should also realize that there are some terms which cannot be defined because no simpler terms exist. Thus a square is a rectangle with four equal sides, a rectangle is a parallelogram with its angles right angles, a parallelogram is a quadrilateral with its opposite sides parallel, a quadrilateral is a polygon of four sides, a polygon is a closed broken line, and a broken line is a series of straight lines; but when one tries to define a straight line, he usually gets a so called definition more involved than the original term, and one which can only be understood if one has a definite idea of a straight line already. Terms such as this are called undefined terms. Our object should be to give the pupil a definite idea or concept of the term but to make no attempt to give a definition of them. Other such terms are point, angle, surface, and the like.

It is also worth while for the teachers to know that defining a term does not prove that it exists. For example, a tangent

to a circle may be defined as a straight line that touches the circle in only one point. This does not assert that there is such a line. Such an assertation can be proved true, however. It is not possible to prove the existence of all defined concepts. We may define a "seven-edged polyhedron" or a "ten-faced regular polyhedron" but it can be shown that neither exists. (44:248) Of course we have no right to use a definition of a supposedly concrete figure in a proof unless we have proved, or assumed that the thing defined exists.

5. *Use of exercises.* a. *Developmental exercises* belong to one of the two types of exercises used in this unit of instruction. These exercises comprise an important part of the unit. "The exercises in the developmental groups come before the statement of a truth and lead gradually to it. They are extremely important and must be considered in the order given."* (43:ix). These exercises approach the problems and theorems by graded steps in order that the pupils may work up to the more difficult problems without the continued aid and guidance of the teacher. These exercises should be used carefully and consistently to obtain the full benefit of the unit. Otherwise the pupil is apt to learn or memorize the proofs and merely repeat them and gain little in the ability to reason for himself.

b. *Practice exercises* are placed immediately after the statement of a definition or the proof of a theorem and are intended to give the minimum amount of practice necessary to give the pupil a clear grasp of its meaning and use. They are also very important, for they give the pupil "a wide range of experience with the particular situation" before he is called upon to use it in the more complex situation of other propositions.

6. *Teaching the analysis of figures.* Pupils often fail on a problem in geometry because they are unable to analyze the figure and see its essential relationships. For example, a pupil may be able to bisect a line segment when it is in a horizontal position and not know where to start when the line is in any other position. Or he may be unable to see that any side of a

* R. R. Smith, Beginners' Geometry. By permission of The Macmillan Company, publishers.

triangle may be considered the base. Recognizing that such conditions exist, or may exist, is but the first step caring for the situation. The teacher should do all in his power to make it possible for the pupil to see such relationships.

The following suggestions may aid in doing this. *a.* Probably the most important thing is the one mentioned above, that the teacher be cognizant of the problem and constantly on the alert to see that the pupil understands each figure that he uses. *b.* The analysis of figures should start early in the work of geometry. For example, when an altitude of a triangle is drawn, get the pupil to see that he is merely drawing a perpendicular from a point to a line, and that the altitude may be drawn from any vertex. *c.* Devices may be valuable in getting the pupil to see the various parts of figures. Colored chalk may be used to set out the part in question, or parts may be drawn separately at one side of the figure as in the case of overlapping triangles. Circular blackboards are sometimes helpful in getting the pupils to see how a part of a figure appears in other positions. These devices should not be continued too long, however.

7. *Teaching the meaning of "Given" and "Prove"* by the use of construction exercises and problems is one of the distinctive advantages of teaching the introductory chapter. If this can be done while dealing with concrete material, it is certainly an advantage to do so. Consider the two exercises; (1) "Construct a triangle with two equal angles and measure the sides," and (2) "Construct a triangle with two equal sides and measure the angles." In the first exercise constructing the two angles equal make the two sides equal, and in the second exercise constructing the two sides equal make the two angles equal. The result is determined by the construction work done. "If the sides are constructed equal—the hypothesis, then the angles turn out equal—the conclusion." The person doing the construction has no control over that part. These are exercises taken from the text of the unit. Numerous other exercises of the same sort make it possible for the teacher to drive the idea home while still dealing with concrete material. The pupil must see that exercises "with different hypotheses must be

constructed differently even though the figures may look alike when they are finished."* (13:324) The pupil should be able to distinguish between hypothesis and conclusion before he takes up the formal proofs of propositions.

II. Meaning of a Demonstration

1. *Teaching the meaning of a demonstration.* a. *Pupils attitude.* Before pupils can adequately learn the meaning of a demonstration, they must want to demonstrate problems. "Until it (the 'urge' to prove a proposition) comes, however, the pupil can profit but little by the study of demonstrative geometry." (29:11) "If he can be brought to see that demonstration gives, where it can be applied, a finality and completeness which must be lacking in the experimental method, he is ready to study demonstration. Otherwise I do not think he is." (2:124)

The awakening of such an "urge" in every pupil cannot be accomplished in any definite way. "To some minds this 'urge' comes early; to others it comes late; to a few it comes not at all." (29:11) How can the pupil be made to see the need of proving or deducing statements and given the "urge" to do so himself? Four things intended to accomplish this have been held in mind while making this unit. They will be discussed below.

(1) The teacher should discuss the method of thinking exemplified by geometry, pointing out that the main aim of the study of geometry is to learn to think in this way and to prove statements by this method. Geometry then becomes a 'game' which one learns to play according to certain rules. All clear thinking is done according to similar rules. The rules are, of course, to set up certain definitions and postulates and to prove, demonstrate, or deduce other statements from these, using pure logic in the argument.

(2) The teacher should call attention to statements or theorems that are not readily admitted to be true by the methods of informal geometry. Even the Pythagorean Theorem on

* From Hassler & Smith, Op. cit. By permission of The Macmillan Company, publishers.

close scrutiny will seem obvious for only a few special cases. Pupils will readily admit after the work of informal geometry that the sum of the angles of a triangle is 180°, but the fact that the sum of the angles of a quadrilateral is 360° is not so easily seen. It is easy enough for a seventh or eighth grade pupil to reason it out, however. (34:42) Attention should be called to these theorems throughout the early part of the course in geometry in order to gradually build up the idea that not all statements can be proved experimentally.

(3) Showing why things are not always what they seem. This is done by presenting some of the familiar optical illusions found in many text books and also by recalling that measuring anything is always inaccurate. They should have learned of the inaccuracy of measurement earlier in their mathematical work; but if they have not, a few simple trials, similar to the one suggested in this unit, will show that measurements are usually inaccurate. In contrast to these illusions and this inaccuracy we may hope to have the pupils eventually see the beauty of a mathematical deduction, which is independent of either faulty observation or inaccurate measurement or construction. Little of this sort can be hoped for in the early part of the study of geometry. It can scarcely be expected earlier than the end of the tenth year.

(4) A frank admission that the primary motive of teaching demonstrative geometry is not to convince the pupils that the theorems are true, but to present a method of thinking. This permits us to include very simple theorems, the truth of which the pupils do not doubt, but which are easily deduced. Geometry then becomes a "game" as was mentioned above, which the pupil can play if he learns the rules. The object in learning the game is that similar rules are used in all clear thinking and they are worth knowing. In time we may hope that the pupil will want to prove the simple theorems, not because anybody ever doubted them, but because they are "a part of the logical chain of geometry."

It must be admitted that such an attitude on the part of the pupil can hardly be expected in its entirety in the early part of the course. It is a desirable aim, however. Full at-

tainment of this aim should be expected at the end of demonstrative geometry rather than at the beginning.

b. *Bringing mental processes to consciousness.* Most pupils by the time they have reached the end of the ninth grade have had occasion to make many inferences. These have been concerned with concrete things, however, and have in most cases been made unconsciously. They have, for example, heard a loud motor and seen people rushing into the street or open spaces and looking upward, and inferred that a zepplin or some other interesting aeronautical machine was passing. The reasoning has not been consciously done, however. They have not said or thought, "Aircraft motors usually make a loud noise; hence this must be some sort of an aircraft passing. But people do not usually rush out to see an ordinary airplane; hence this must be a zepplin or some peculiar sort of an airplane." The conclusion has been reached without knowing just what the thought process was.

c. *Making simple inferences.* The object of the first part of this chapter will be to bring this thought process to consciousness and to learn to apply this logical form to algebraic and geometric situations. Much practice in making simple inferences is given in the text of the unit. This practice overcomes one of the main difficulties of beginning demonstrative geometry with theorems in sequence. The difficulty is that theorems cannot be graded in such a way that the pupils can learn the numerous elements gradually.

Enough exercises are included in which the figures are not drawn to show the pupil that the thought process is independent of the figure, and to prevent his saying that certain lines or angles are equal because they look equal. The object has been to give the pupils "simple exercises which are near enough alike so that the doing of one will help in the doing of the next, but sufficiently different so that mere memorization will be a hindrance rather than a help."* (13:330)

2. *The use of definitions, axioms, and postulates.* Definitions, axioms, and postulates, along with the undefined terms,

* From Hassler & Smith, Op. cit. By permission of The Macmillian Company, publishers.

constitute the ground work or foundation of any geometric sequence. If we desire to make a perfectly logical science the postulates must be independent, that is, it must not be possible to deduce one of them from another. Since we are not interested in building a strictly logical sequence, no attempt is made to make the assumptions of this unit independent. In any case, however, it is necessary that they be non-contradictory and that they exactly define the science. "Herein lies one of the good arguments for a little demonstrational geometry in grade nine, where we can begin wherever it is most convenient and make such assumptions as we choose, and select for proof the facts that are best suited to our purpose." (2:127) In this unit we have chosen to define only the terms we need and express only such postulates as are necessary to prove the few propositions included. We have assumed many propositions (after experimental development) necessary in the sequence but for which it is undesirable to give a proof.

3. *The use of informal and formal proofs.* All of the early proofs of this unit have been made informal for the reason that the most desirable thing for the pupil to get is assumed to be an understanding of the reasoning process. This comes most naturally in the language with which the pupil is most familiar. Formal proofs are included and required for many of the theorems and exercises in this unit, but the teacher must be sure that the pupil has a definite understanding of the proof before anything in regard to form of proof is required.

Some doubt has been expressed in regard to the use of formal proof in the ninth grade work. To quote Barber:

"The ninth grade work in demonstration will probably be most successful if pupils are not required to reproduce the proofs unassisted, but to be able to give the general plan of a proof or to supplement it by giving reasons for the steps of the argument." (2:126)

It seems reasonable to suppose, however, that at least some of the pupils will be able to profit by the use of the formal proof.

4. *The congruence theorems and the exercises following them.* The congruence theorems are treated only experimen-

tally. There seems to be no reason whatever for introducing superposition in the ninth grade. After experimental constructions and comparisons the pupils are allowed to assume the first two congruence theorems; that is, "If two triangles have two sides and the included angle of one equal respectively to two sides and the included angle of the other, they are congruent," and "If two triangles have one side and two adjacent angles of one equal respectively to one side and two adjacent angles of the other, they are congruent." Attention should be called to the two phrases, "two sides and the included angle" and "one side and two adjacent angles." A surprisingly large percentage of pupils have trouble with these phrases and a considerable amount of practice is given in this unit to overcome this difficulty. The teacher should pay especial attention to these exercises. Numerous exercises using the facts assumed in these two theorems may be constructed. If these are graded properly and presented carefully, they will go a long way toward showing the pupils the meaning of demonstration.

III. A Sequence of Theorems

The methods of teaching these theorems and exercises will be the same as for those discussed above. Some theorems will be proved experimentally, some informally, and some formally. Exercises will be used in the same way as in the other theorems and problems.

The sequence of theorems presented are not based on a minimum number of postulates or assumptions; on the other hand theorems are assumed when the proof would be too long or too difficult or both, and when too many theorems would have to be proved to get to the one wanted. An example of the last case would be the theorem regarding the equality of the alternate interior angles of parallel lines. This makes it possible to include the theorem about the angle sum of the triangle and the Pythagorean Theorems without making the sequence too long.

It is desirable that the pupil see the significance of the sequence and the interdependency of the theorems and postulates. It is not always possible to accomplish this, however, especially on such a short exposure.

Section VII CORRELATIONS

A. With other subjects.

1. *With science.* The statement and proof of a scientific law follows the same steps as discovery and proof in geometry; first, experimentation and attempts to make a general statement to cover the facts discovered; (similar to informal geometry), second, an attempt to prove that the general statement is true, the proof to be based on the facts already known to be true, or assumed to be true. (similar to demonstrative geometry) (34:41-42)
2. *With debating or other types of argumentation.* All good argument should be based on the same logical principles as a demonstration in geometry.
3. *With manual arts and mechanical drawing.* The constructions in the early part of this unit aid in designing patterns for many kinds of articles made in the shops. Also in much of mechanical drawing the figures are identical with these.

B. Within the subject.

1. *With informal geometry.* Most if not all of the propositions discussed in this unit are proved experimentally in informal geometry. Compare the two methods both as to form and value. Let the pupils decide which is the most convincing. Bear in mind, however, that demonstrative geometry is not taught in order to convince the pupils of the truth of the propositions.
2. *With algebra.* Algebra is used in most of the proofs of demonstrative geometry. In these simpler theorems it is limited to the use of symbolism and the use of algebraic axioms. Further on in geometry, however, "many a geometric problem permits its query to be formulated in algebraic language and the solution to be found by the processes of algebra." (60:276) The Pythagorean relation is one fact which may be used often in this way.
3. *With trigonometry.* Three parts of elementary trigonometry are connected with geometry; the fundamental ratios of the right triangle, the formulas for the solution of the general triangle, and the fundamental relation between the ratios. These three things are essentially geometric and might be included in the treatment of geometry. (60:228) If some indirect measurement has been studied, the ratios,

tangent and cosine correlate directly with similar triangles as studied in this unit.

C. Within the unit.

1. *The relation between postulates and theorems.* Demonstration of a group of theorems presupposes definite relations between the theorems and between the fundamental assumptions and the theorems. This is one of the things we hope the pupil will get from the unit.
2. *Relation between the different kinds of proof.* The pupil should see the relation between the different kinds of proof, that is, the experimental proof, the informal proof, and the formal proof. He should also see the essential differences.
3. *The relation between "given" and "to prove"* should be clear in the pupil's mind.
4. *The connection between the analytic method of proof and the synthetic method of proof* should be clear to the better pupil.

Section VIII. BIBLIOGRAPHY

A. For teachers.

Austin, W. A. A Laboratory Plane Geometry. Scott, Foresman Company, 1927.

In this book the pupil is expected to construct plates containing most of the figures used in plane geometry. The pupils also write out the proof. The class recitation is also on the laboratory plan.

Barber, H. C. Teaching Junior High School Mathematics. Houghton Mifflin Company, 1924.

This book contains a brief discussion of the reasons for and the methods of teaching demonstrative geometry in the junior high school. Also there is a good discussion of the teaching of junior high school mathematics in general.

Clark, J. R. and Otis, A. S. Modern Plane Geometry. World Book Company, 1927.

The introduction of geometry in this book uses the idea of symmetry. From this idea the authors show reasons for wanting to draw perpendiculars to lines, constructing angles equal to given angles, and the like.

Hassler, J. O. and Smith, R. R. The Teaching of Secondary Mathematics. The Macmillan Company, 1930.

This book has an excellent discussion of the fundamental concepts of mathematics and of the method of introducing demonstration. Professor Smith's book

Beginners' Geometry was used extensively in selecting the subject matter for this unit.

National Committee on Mathematical Requirements, Report of. The Reorganization of Mathematics in Secondary Education. The Mathematical Association of America, 1923.

This well known report makes an optional recommendation that some demonstration be included in the ninth grade of the junior high school course. It is because of this recommendation that most of the work along this line has been done.

National Council of Teachers of Mathematics, Fifth Yearbook. The Teaching of Geometry. Teachers College Bureau of Publications, 1930.

This book contains several articles on the teaching of demonstrative geometry in the junior high school. All of the other articles deal with some phase of the teaching of geometry.

Sanford, Vera. A Short History of Mathematics. Houghton Mifflin Company, 1930.

This book is a good brief history of mathematics. it contains much interesting material about the origin of geometry as well as a discussion of particular problems.

Smith, D. E. and Reeve, W. D. The teaching of Junior High School Mathematics. Ginn and Company, 1927.

This book contains an excellent chapter on the teaching of demonstrative geometry as well as a discussion of the general junior high school situation in regard to mathematics.

Smith, R. R. Beginners' Geometry. The Macmillan Company, 1929.

This book contains much of the subject matter for this unit. Though it is a tenth grade book, the introductory chapters serve excellently for ninth grade geometry.

Stone, J. C. and Mallory, V. G. Modern Plane Geometry. Benj. H. Sanborn & Company, 1929.

This book contains many practical applications of geometry. Many of them may be solved by the theorems proved in this unit.

Young, J. W. A. The Teaching of Mathematics in Elementary and Secondary School. Longmans, Green and Company, 1924.

This is probably the most complete American book on the teaching of secondary mathematics. It con-

tains many helpful suggestions on the teaching of geometry.

B. For pupils.

Clark, J. R. and Otis, A. S. Modern Plane Geometry. pp. 1-50.

Smith, D. E. and Reeve, W. D. Essentials of General Mathematics, Book II.

Sanford, Vera. A Short History of Mathematics.

Strader, W. W. and Roads, L. D. Plane Geometry. pp. 1-65.

Section IX. TESTING PROGRAM

Pretest (43:26)*

(No time limit)

1. Read the angle in Fig. 1, using three letters for each, for example, $\triangle 2 = \triangle DBC$.

2. Draw a triangle, bisect the three sides and connect the points of bisection with the opposite verticles.

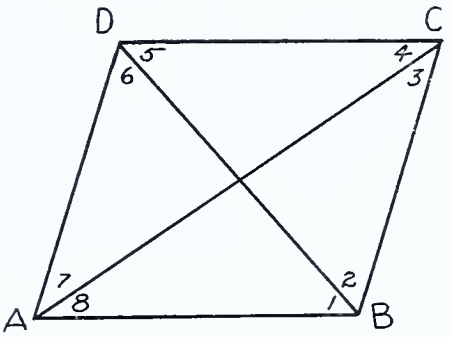


FIG. 1

3. Draw a triangle, bisect each angle, and from the points of intersection of the bisectors construct a perpendicular to each side of the triangle.
4. Construct a triangle ABC with angle B equal to angle C. Compare AB and AC with compasses. What parts of this triangle are constructed equal? What parts *turned out* to be equal?
5. Construct an isosceles triangle and bisect the vertical angle.
6. In Fig. 2. AB and DE are both perpendicular to BD. C is the middle point of BD. Construct the figure. Measure AB and DE, and AC and CE with compasses. What parts of this figure were constructed equal? What parts *turned out* to be equal?

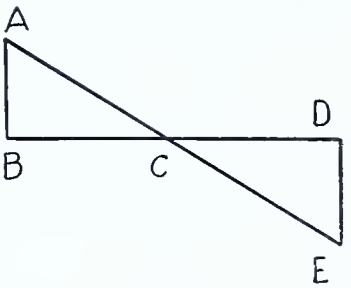


FIG. 2

* From R. R. Smith, Op. cit. By permission of The Macmillan Company, publishers.

7. In Fig. 2 AB and DE are constructed perpendicular to BD, and AB = DE. Construct the figure. Measure AC and CE, and BC and CD with compasses. What parts of this figure were constructed equal? What parts *turned out* to be equal?

8. In Fig. 3 $\angle 1 = \angle 2$ and AB=CD. Construct the figure.

9. In Fig. 3 $\angle 1 = \angle 2$, and $\angle 3 = \angle 4$. Construct the figure.

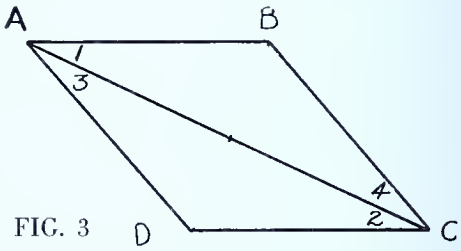


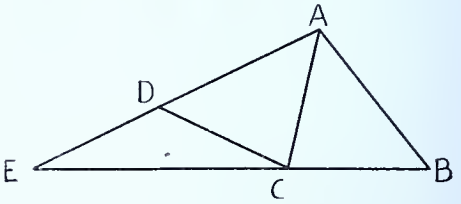
FIG. 3

10. Bisect an angle and tell, in good geometric form, how you did it.

Test I

1. A geometric statement whose truth is accepted without proof is called _____ _____.
2. A statement to be proved is called a_____.
3. One of the two adjacent angles formed when one line is perpendicular to another is called_____ _____.
4. If the sum of two angles is equal to two right angles (180°), the angles are said to be_____.
5. If two sides of a triangle are equal it is called_____ _____triangle.
6. An angle made by two lines meeting at a point is said to be_____between the two lines.

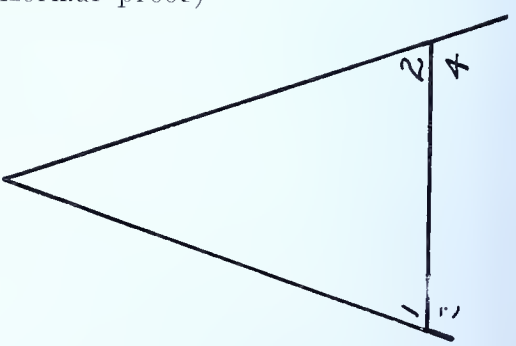
7. In the figure at the right AC=DE, BC=CD, and CD=DE. Prove that the triangle ABC is isosceles.



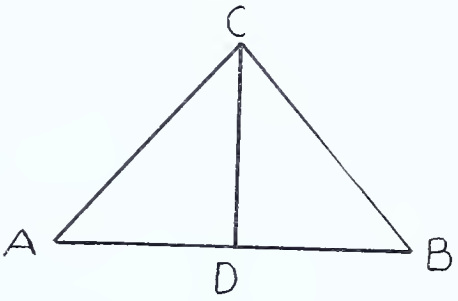
8. If two angles are supplementary to the same angle, they are equal. (give an informal proof)

9. If $\angle 1 = \angle 2$ in the figure at the right, prove that $\angle 3 = \angle 4$.

(State the "given" and "to prove" and give the proof in good geometric form.)



10. Give a formal proof,
Given: CD bisects $\angle$ C.
and AC = BC.
To prove: $\angle$ A = $\angle$ B.



Test II. Unit Achievement Test

1. If from any point on the bisector of an angle lines are drawn making equal angles with the bisector, they cut off equal distances on the side of the bisector.
(Draw the figure and state the "given" and "to prove".)
2. Lines drawn from any point on the perpendicular of a line to the extremities of the line form an angle which is bisected by the perpendicular. (give a formal proof)
3. If two straight lines are cut by a transversal so that the alternate interior angles are equal, the lines are parallel.
4. The sum of the angles of a triangle is equal to two right angles.

CHAPTER VI

Summary and Conclusions

Object of the Study

The purpose of this study as stated on page 1 has been to determine a method of improving instruction through the use of the teaching plan of a unit in the field of junior high school mathematics, and to test the value of the teaching plan of a subject matter unit in actual classroom situations.

Methods of Procedure

1. Various types of units of work have been discussed and defined.

Units of work vary from most formal listing of subject matter set-up-to-be-learned to those units in which the child does that in which he is most immediately interested. Different schools have used these various kinds of units of work in the past few years as a basis for curriculum construction.

Many attempts to integrate the curriculum or to build integrated units of work in which it is hoped the experiences of the child may be organized into a united whole so that the pupils may make appropriate responses have been made. The best work along this line has been done by combining or integrating subjects such as social science, with English, art and the like. Units of correlated subject matter, in which the subject is helped and amplified by subject matter from any other field, have also been used in building an integrated curriculum. Such units appear in the course in junior high school mathematics.

2. The teaching plan of a unit has been defined and discussed.

A pedagogical unit, or correlated unit, as it has been called in this study, in itself presupposes some kind of teaching plan. The plan in the mind of the textbook writer may not transfer to the teacher, however. A teacher may teach such a unit from

day to day without any plan, he may have a fairly definite plan in mind, or he may have a definitely written plan. This written plan may take many different forms and may be brief or elaborate. The teaching plan submitted in this study is quite elaborate and makes an attempt to cover most, if not all, of the details of teaching a unit.

Therefore, the teaching plan means something more than the textbook writers outline. Many things that will not appear in the textbook are included in the scheme of the artist teacher. These include, among others, the unity and extent of the topic being taught, a knowledge of the pupils in the class, and a definite testing program for the entire unit.

To teach mathematics and other subjects efficiently requires careful planning. This is especially true of the reorganized mathematics of the seventh, eighth, and ninth grades which usually includes subject matter from informal geometry, social-economic arithmetic, algebra, trigonometry, and in some cases demonstrative geometry. Much of this work is new or organized in ways different from those of the old type curriculum. The teacher is more or less unfamiliar with the mathematical work of these grades because of the new material and also because of the reorganization of the old material. Teaching plans are included illustrating all of these types of subject matter.

3. To determine the use made of the various items of the teaching plan by experienced teachers, questionaires dealing with the way the different items of the plan were used were sent to teachers scattered over the United States. Usable replies were received from one hundred seventy-seven teachers in ten different states.

4. To determine the value of the teaching plan in actual classroom situations, four of the plans were used experimentally.

Selection of the Unit to be Used

The units to be used were selected with three things in mind. Two were, to include units from different grades of the junior high school, and to include the so called formal or stereotyped units of work, represented by Teaching Plan No. IV on

Square Root, Radicals, and Exponents, as well as the newer types of units, represented by Teaching Plan No. I on *Insurance* and by Teaching Plan No. II on *How to use Equations*, and the like. The third thing considered was that the unit used be fairly well organized in the course of study of the school in which it was taught.

Selection of the Teachers and Pupils for the Groups

The object in choosing the teachers and pupils was to get a random selection. For this reason units were taught in a number of different schools and classrooms. In all, eighteen different schools taught units using the teaching plan. Some of these had several classes using them. The teachers were chosen on the basis of willingness to assist. In most cases the teachers and pupils of the control groups knew nothing of the experiment.

Statistical Procedure

The two group method of comparison was used. The two groups were equated according to intelligence and were then taught identical subject matter. One group of teachers (the experimental group) used the teaching plan of the unit; the other group of teachers (the control group) taught by their usual method. The same achievement tests were given to both groups of pupils, and the marks made on these tests were taken as the measure of their achievement on the unit. It was assumed that, since the groups were equal in intelligence, and had had their previous work in the same school system, they were equal from the standpoint of mathematical ability.

The means, medians, and standard deviations of the scores on both intelligence and achievement were computed. The actual differences of these measures were then computed, together with the probable errors of the differences. These probable errors of the differences were used in determining whether or not the actual differences were significant.

Results and Interpretations

The questionaire regarding the use made by experienced teachers of the items of the teaching plan showed that a large majority made some use of each item included. In most cases less than half of them had these items of their "plan" written out. Only a small number of teachers declared that they failed to make use of these items. A large majority (116 to 46) of these teachers said they believed it valuable for the experienced teachers to have these items written out in their teaching plan, and an overwhelming majority (168 to 8) declared that they believed it valuable for an inexperienced teacher to have them written out.

These results seem to indicate that at least these items are of importance in planning the teaching of any unit. The opinions of the teachers also seem to indicate that a teacher should have a written teaching plan in his hands if the best possible teaching is to be done.

In the experimental work with the teaching plan of a unit the experimental groups made the best scores on the achievement tests for each of the four units used. In three of these units the differences of the means and medians were greater than four times the probable error of the differences; this indicates that they were highly significant differences. In the other case the differences of the means and medians were greater than three times the probable error of the differences; this is an indication that while the differences could not be attributed to the fluctuations of sampling, they could not be interpreted as highly significant.

These results seem to indicate that the use of the teaching plan in actual classroom situations results in better learning of the type necessary to succeed on the tests given. Care must be taken, however, to consider other factors that are beyond the control of an experiment of this sort, as, for example, the varying ability of teachers to teach pupils in such a way that they will be able to succeed on a certain type of test, the general attitude of the pupils, and the like.

Recommendations

1. The teaching plan of a unit may be used to raise the various phases of the teacher training program to a functional level. An actual situation and a hypothetical situation have been described (Chapter IV) with the reactions of the administrators, teachers, and student teachers.

2. The teaching plan of a unit would probably serve to stimulate better teaching on the part of teachers in service as well. It is recommended that those interested in raising the professional standards of teachers-in-service consider the making and using of teaching plans as one sort of teacher activity that may aid in accomplishing the end in view.

3. Little or no work has been done in making actual teaching plans for course of study units, and including them in the course of study for the use and guidance of teachers. The teaching plan proposed in this study is submitted as one of the ways in which classroom activities may be planned for inclusion in courses of study.

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